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Routine Mathematical Problems and Mathematical Inquiry in an Elementary Classroom: Tensions and Struggles

by

Dianne Joyce Dodsworth ©

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Department of Elementary Education

Edmonton, Alberta

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University Of Alberta

Faculty of Graduate Studies and Research

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled *Routine Mathematical Problems and Mathematical Inquiry in an Elementary Classroom: Tensions and Struggles* submitted by Dianne Joyce Dodsworth in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Dedication

This study is dedicated to the memory of my father, Harold Lynch, my aunt and uncle, Joyce and Joseph Sheehan and my two grandmothers, Catherine Dodsworth and Dora Seyforth.

By their example, these members of my family have encouraged me to persevere and to strive to be the best in what I do.

Abstract

This study addresses the tension between the NCTM's focus on problem solving and teachers' efforts to develop mathematically meaningful problems. The *Standards* [1989] state that if a mathematics program is to be consistent with its goals, the problem solving approaches should be compatible with its vision. The study explores the nature of mathematical inquiry in a classroom and the relationship between this approach and the problem solving practices of the students.

Two sets of data were collected for the study. One set of data was collected from a selected elementary classroom in order to explore the nature of what the teacher described as an inquiry-oriented approach to mathematics. The analysis of this data revealed a wide range of teacher practices. Included in the teaching practices and relevant to this study is the teacher's practice of presenting routine problems to her students. Possible reasons for the inquiry-oriented teacher continuing to present routine problems are presented.

A second set of data was collected from audio tapes of four Grade 3/4 students as they worked on routine problems. The analysis of this set of data revealed five problem solving practices. The data suggested that, when presented with routine problems, the students, despite learning mathematical problem solving in what the teacher described as an inquiry-oriented approach, appeared to choose a "number oriented" style of problem solving.

The study concludes that the students' problem solving was affected by the nature of the routine problems and students' embedded beliefs about mathematics. The study also concludes that such classroom teaching practices as modeling problem solving strategies, presenting mathematics as

a linear process and presenting routine problems possibly affected the students' problem solving practices. The findings highlight the tensions teachers face as they struggle to enact mathematically meaningful problem situations.

Implications of the study include suggestions for further research on the multi-layered phenomena of "good" mathematics teachers and on teachers' embedded beliefs about "doing mathematics". Other implications for research are on how societal perceptions of mathematics affect teacher's mathematical practice.

Acknowledgments

This study has been completed through the on-going support of friends, family and academic advisors. I would first like to acknowledge my thesis advisor, Dr. Roberta McKay, Department of Elementary Education, University of Alberta, for her positive and encouraging support throughout the past years of my program.. I also acknowledge Dr. Daiyo Sawada for his comments and suggestions through which our ideas slowly and often painfully, became reality. I also acknowledge the members of my thesis committee as well as members of the Department of Education at the University of Alberta. As I increasingly became a "fixture" on the 5th floor of the Education Building at the University, I felt nothing but support and encouragement as I worked through my Ph. D. program.

I would like to acknowledge the consistent encouragement of my friends and family. I would first like to acknowledge the financial and emotional support of my mother, Dorothy Lynch, without whose initial and on-going help, this doctorate would not have been started or completed! I also acknowledge my daughter, Adrienne Jennifer Allen and my son, David Allen who seemed to know just when to ask how "things were going" and when to say little or nothing! Acknowledgments are also given to Samantha, Kristina and Kimberly Turner for being so understanding when I was required to work at my computer on the "project" rather than joining them in family activities. I would like to acknowledge Hart Mueller for his editing suggestions, Nora Jean Paradis for providing me with a "home base' to work from when at the University of Alberta.

Finally, I wish to shower my appreciation on my partner, colleague and friend, Jeff Turner. Jeff's constant emotional, professional and logistical help was crucial to the completion of my Ph. D. His greatest contribution has been his patient listening, supportive suggestions and attending to the daily details of living so that I was able to concentrate on the completion of this study.

To all of these people my heartfelt thanks for allowing me to attain my goal!

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CHAPTER I

INTRODUCTION

This chapter presents the purpose and rationale of the study, the research question and an overview of the study design. The chapter begins by presenting the study in the context of the past twenty years of research in mathematics.

In the last twenty years, a number of reports have identified shortcomings in the present mathematics curriculum and its teaching. Examples of such reports include: *An Agenda for Action: Recommendations for School Mathematics in the 1980's* [National Council of Teachers of Mathematics, 1980]; *A Nation at Risk* [NCEE, 1983]; *The Underachieving Curriculum: Assessing US School Mathematics from an International Perspective* [McKnight, Crosswhite, Dossey, Kifer, Swafford, Travers & Cooney, 1987]; *Curriculum and Evaluation Standards for School Mathematics* [NCTM, 1989]; *Everyone Counts* [NRC, 1989]; *Reshaping School Mathematics* [NRC, 1990]; *Professional Standards for Teaching Mathematics* [NCTM, 1991] and *Assessment Standards for School Mathematics* [1995]. The shortcomings, identified in these reports, are a consequence of the social and economic changes in the world which have fostered a strong public awareness of the transition from the "industrial age" to the "information age" [NRC, 1990].

As society enters this new informational age, changes are taking place in the workplace, the home and daily life. Information has become the new "raw material" and the abilities to interpret and communicate information are necessary skills [Steen, 1986; NCTM, 1989; NRC, 1990].

Technology has opened new frontiers for new mathematical applications and has made mathematics essential for helping young people cope in the modern world [NCTM, 1989].

Many educators have perceived mathematics as being a cold and austere discipline which provides little scope for judgment or creativity. These views are being challenged by a growing number of philosophers of mathematics [Davis & Hersh, 1981; Kitcher, 1988; Lakotas, 1976; Tymoczko, 1986]. They argue that mathematics is illusive, fluid, and, like any other body of knowledge, the product of human inventiveness. Mathematics, from the perspective of these writers, has many facets. It can be seen as a language, as a kind of reasonable structure, as a collection of knowledge about numbers and space, as an arrangement of methods for deriving conclusions, as the core of the understanding of the physical world or as an engaging intellectual activity.

This view of mathematics has had an impact on education. In 1986 the Board of Directors of the National Council of Teachers of Mathematics established the Commission on Standards for School Mathematics as one means to help improve the quality of school mathematics. The establishment of this commission reflected a major rethinking about what should happen in mathematics classrooms. This rethinking was framed by the idea that mathematics is created by us and that it can make sense. This is a significant change from perceptions that mathematics is a set of rules to be learned and practiced.

The National Council of Teachers of Mathematics has responded to the emerging challenges for teaching and learning mathematics by publishing several documents including: *Curriculum and Evaluation Standards* [1989]; *Professional Standards for Teaching Mathematics*

[NCTM, 1991] and *Assessment Standards for School Mathematics* [NCTM, 1995]. These documents were published in order to reflect and extend "the community's responses to the demands for change" [*Standards* 1989, p. 1]. Inherent in these documents, as stated in the *Standards* [1989], is the NCTM vision that "all students need to learn more, and often different, mathematics and that all instruction in mathematics must be significantly revised" [*Standards*, p. 1].

The *Curriculum and Evaluation Standards for School Mathematics* [NCTM, 1989], which will be referred to from this point on as the *Standards* [1989], is based on reactions to a working draft of the document gathered during the 1987-1988 school year and drafted during the summer of 1988 by four groups, each representing a cross section of mathematics educators. Included were classroom teachers, supervisors, educational researchers, teacher educators and university mathematicians. The resulting *Standards* [1989] is a document designed to establish a broad framework to guide reform in school mathematics. It presents a vision of what the mathematics curriculum should include in terms of content, priority and emphasis to provide equal opportunities for all children to become lifelong learners, informed citizens and mathematically literate consumers and workers. To reflect the importance of these societal goals and of mathematical literacy, the *Standards* [1989] suggest that new school mathematics programs must ensure that children learn to value and enjoy mathematics, become confident in their abilities to do mathematics, become mathematical problem solvers, learn to communicate mathematically and learn to reason mathematically. These recommendations of the National Council of Teachers of Mathematics, specifically those goals that address mathematical problem solving, provide a context for this study.

From a personal perspective, I observed the need for the changes suggested by the National Council of Teachers of Mathematics when I taught Grade IV Mathematics. I observed that my students became more passive and appeared to be less engaged when they moved from the more active approach which I used in Language Arts to the more traditional approach in Mathematics. In response to my students' needs for a more active approach, I moved the focus in my mathematics program to problem solving, specifically to problems generated by the students. I was surprised and encouraged by the change in the attitude and the learning of my students. They were more engaged in this new approach and there appeared to be an improvement in their learning. When I read the *Standards* [1989], I felt supported in my new approach to Mathematics. From my experience, I became interested in the NCTM's focus on problem solving in mathematics education.

The NCTM suggests that classrooms aligned with its vision of mathematics teaching and learning are places of inquiry where interesting problems are regularly explored using important mathematical ideas. Mathematics educators have argued for years that an inquiry-oriented approach, with its emphasis on thinking strategies, can improve student learning. An inquiry-oriented approach to the teaching and learning of mathematics and mathematical problem solving stimulates students to use their own natural ways of thinking in order to generate a variety of answers or approaches to solving problems, to present their solutions to the class, to participate in discussion of the mathematical quality of different solutions, to search for interesting methods of reaching solutions and to formulate or pose related problems. The search for different possible solutions to a problem encourages students to explore, discover, analyze,

verify, generalize, discuss and, generally, view mathematics as an exciting discipline of study [Hashimoto, 1988; Nagasaki & Becker, 1993; Becker, 1993]. The NCTM vision for mathematical problem solving includes: [1] the ability to set up problems with the appropriate operations; [2] knowledge of a variety of techniques to approach and work on problems; [3] understanding of the underlying mathematical features of a problem; [4] the ability to work with others on a problem; [5] the ability to see the applicability of mathematical ideas to common and complex problems; [6] preparation for open problem situations; and [7] belief in the utility and value of mathematics [*Standards* , 1989, p. 4].

The advantages of an inquiry-oriented approach to mathematics and mathematical problem solving are that students actively participate in the lessons, express their ideas freely and frequently and have more opportunities to make use of their mathematical skills. Such an approach encourages students to be intrinsically motivated, to use reasoning, to have rich experiences in the enjoyment of discovery and to receive the approval of their peers. Students are given the opportunity to respond to the problems in some significant way, regardless of their abilities [Becker, 1992; Becker & Shimada, 1993; Becker, 1993].

The challenges for educators are to move mathematics teaching and learning and mathematical problem solving away from a traditional perspective. In such a perspective, students learn mathematics at a rote level, without understanding how to apply what they know, to problem solving situations [NCTM, 1980; NCTM, 1983; McKnight et al, 1987; NCTM, 1989; Crosswhite, 1990; NRC, 1990; Romberg, 1990; NCTM, 1991; Kamii, 1994]. Teachers often observe this perspective in classrooms when students move quickly to solve a problem without first thinking about

the information and data available in the problem. These students arrive at answers that are not even remotely possible as solutions or they insist that they have no possible means to attempt a solution to a problem. Teachers whose teaching and learning of mathematics is guided by the vision and recommendations of the *Standards* [1989, 1991, 1995] may be perplexed by the students' problem solving practices even as they struggle with their own inadequate experiences with problem solving and lack of a proper pedagogy to teach problem solving [Stonecipher, 1986; McKnight et al, 1987; Becker, 1990; NCTM, 1991]. It is often difficult for teachers to develop mathematically meaningful problem situations and successfully pose problems [Nagasaki & Hashimoto, 1984].

This study addresses the issues of students' mathematical problem-solving practices in relation to the nature of teaching and learning of mathematics and mathematical problem solving.

Purpose of the Study

The purpose of this study is to examine and address the tension between the NCTM's major focus on problem solving and teachers' efforts to develop mathematically-meaningful problem situations [Nagasaki & Hashimoto, 1984]. The *Standards* [1989] state that if a mathematics program is to be consistent with its goals, objectives, mathematical content, and topic emphases, the mathematical problem-solving approaches, materials and activities should be compatible with its vision and intent [*Standards* 1989, p. 241]. The *Standards* [1989] states that "What a student learns depends on how he or she learns it" [p. 6].

This study explores the "what": that is, the teaching and learning of mathematics and mathematical problem solving in what has been described

by the classroom teacher as an inquiry-oriented approach and the "how": that is, how presenting students from this classroom with traditional, routine problems affect their problem solving practices. It has been noted earlier in this chapter that a disadvantage to an inquiry-oriented approach to mathematics and mathematical problem solving is that teachers often struggle with the lack of a proper pedagogy for teaching problem solving. Teachers present a variety of mathematical materials and activities to their students and some of these mathematical tasks include traditional, routine problems. Such problems, taken from a textbook or from the teacher's files, are artificially directed to specific number operations and, usually, to a single, correct answer [Bottge & Hasselbring, 1993]. The use of such problems by the teacher illustrates the concern about what students are learning [mathematics and mathematical problem solving in what the teacher describes as an inquiry-oriented approach] and how they are learning [being given routine problems to solve]. This study addresses the relationship between what and how students learn mathematics and mathematical problem solving.

Rationale

This study poses an important question for current mathematics education and for teachers who are using a variety of instructional conditions and materials in their classrooms. If mathematical problem solving is taught from what is described as an inquiry-oriented perspective, where interesting problems are explored using important mathematical ideas, does the type of mathematical problems presented to students - for example routine, traditional problems - affect their mathematical problem solving practices? What practices do these routine, traditional problems

elicit?

The NCTM *Standards* [1989] states that "the spirit and vision of the *Standards* cannot be achieved if instruction is inconsistent with its underlying philosophy. Specifying the content for a quality mathematics program is impossible without addressing the accompanying instructional conditions" [p. 252]. Walkerdine [1988] states that "practices are produced because that is what the event is set up to do" [p. 33]. This study explores the practices elicited from the "event" of solving routine, traditional problems.

Proponents of current curriculum reform argue for a particular perspective, one in which the process as well as the product of students' mathematical learning is emphasized. This perspective often differs from the perspectives of others, including parents and the public. If mathematics learning is seen as a process of inquiry, then it is possible to view it as a continually expanding field of human inquiry which empowers its learners and gives them access to its concepts. This study, with its focus on the process of learning mathematics, specifically problem solving, is nested in this current curriculum reform in mathematics education.

Statement of the Research Question

The question that guides this study is centered in the kinds of practices demonstrated by elementary students when they are presented with routine problems such as those found in most textbooks and commercial worksheets [Bottge & Hasselbring, 1993; Becker, 1983, 1990; Stonecipher, 1986] and of the variety that many teachers may use. Of significance to this study is that these elementary students were learning mathematics in what their teacher described as an inquiry-oriented

approach to mathematics and mathematical problem solving.

Guiding Question: What mathematical practices are elicited when Grade III/IV students, learning mathematics in what is described by their teacher as an inquiry-oriented approach, are presented with routine, traditional mathematical problems?

Overview of the Study Design

Research on the nature of mathematical teaching and learning and mathematical problem solving requires an approach that is qualitative in nature in order to address a complex process that is influenced by a large number of factors [Lester, 1985]. Such an approach values [1] context and setting; [2] meaning and process as crucial in understanding human behavior; [3] the importance of descriptive data recorded in the field; [4] the inductive analysis of data which allows categories and patterns to emerge and [5] a flexibility in the study design [Bogdan and Biklen, 1992]. This study uses a qualitative design in order to address the guiding question of this research.

This study took place over a 10 month period in a Grade III/IV multi-aged classroom. The guiding question of this study suggested two sets of data collections and analyses.

Data were first collected from the classroom, the teacher, the students and the curriculum in order to explore and document the nature of the teaching and learning of mathematics and the alignment of this teaching and learning with the "spirit and vision" of the NCTM *Standards* [1989]. The nature of the teaching and learning of mathematics and mathematical problem solving in the elementary classroom is defined through data

2. One Classroom With One Teacher: This study took place in one classroom with one teacher. Therefore, the findings of this study are specific to this particular setting. This, along with the small sample size, would indicate that caution should be taken in generalizing the findings of this study to another setting.

3. Problem Solving Sessions: The six sessions took place in a setting apart from the classroom. The reason for removing the students from the classroom was to provide for a clear audio-tape sound in the separate setting away from classroom noise and activity. However, this new setting in a corner of the multi-purpose room, appeared to be almost "experimental". Mercer [1995] states that there is a lot of evidence, especially from the study of children's cognitive development, that the performance of subjects in experimental tasks is strongly affected by their interpretation of what they are meant to do. In this research setting, students were not only working on what is described as routine, traditional problems but they were now in a setting that could be described as contrived, almost "test-like". The setting apart from the classroom should be taken into consideration in respect to any generalizations from the study.

4. Gender of the Students: The four students chosen for the problem solving sessions were all girls. Two students were in Grade III and two were in Grade IV. The fact that all four students were girls was not seen as a focus for the study but this factor has the potential to affect the generalizability of the study.

Summary

Chapter I has presented an introduction to the study. Chapter II presents a review of literature relevant to the relationship between the

compiled by observation and recorded in field notes, a dialogue journal with the classroom teacher and classroom artifacts collected over an eight month period in the elementary classroom.

A second set of data were collected during six problem solving sessions. The data were collected from the audio-taped talk of four students, chosen from the selected classroom, as they worked on three sets of routine, traditional mathematical problems. Their work on these problems took place in a setting apart from the classroom and the teacher. Data were compiled from the transcriptions of the audio tapes and field notes. From these data, problem-solving practices were identified and described.

Delimitations of the Study

According to Mauch, J. E & Birch J. W [1998], the delimitations of a study are construed as the boundaries of the study. Examples of delimitations of a study could include such concerns as the nature and size of the study, the uniqueness of the setting and the time period during which the study was conducted.

This study includes the following delimitations:

1. Selection of Problems: The problems for the problem sessions were chosen from the researcher's files. Lynne reviewed the problems to make certain that they were not too difficult for the students. Part of the reason why the problems were chosen was because they were similar to the types of problems that teachers might choose. Such problems appear to address the interests of the students and, therefore, appear to be aligned with the NCTM "vision" for problem solving. The choice of the problems should be a consideration in generalizing the findings of this study.

problem solving practices of the students. Chapter III presents the research methodology for the study. Chapter IV presents the analysis of the classroom data and Chapter V presents the analysis of the problem solving data. Chapter VI presents concludes the study with an overview, discussion of the findings and a summary of the study. Chapter VII presents the findings and implications of the study. Also presented will be a discussion of the study, followed by the conclusions and implications of the study for the teaching and learning of mathematics and mathematical problem solving.

CHAPTER II

REVIEW OF RESEARCH LITERATURE

This chapter presents a review of research relevant to the purpose of this study which is to address concerns regarding the tension between the NCTM's major focus on problem solving and teachers' efforts in developing mathematically meaningful problem situations. The literature is reviewed under the following topics as they pertain to the two sets of data collected for this study: [1] Overview of Literature Relevant to Change in the Teaching and Learning of Mathematics and Mathematical Problem Solving; [2] Literature Relevant to the Classroom Mathematical Inquiry; [3] Literature Relevant to the Classroom Organization for Problem Solving; [4] Literature Relevant to the Classroom Teacher's Role in Problem Solving; [5] Literature Relevant to the Routine Problems Presented in the Problem Solving Sessions [6] Literature Relevant to Defining Problem Solving for the Classroom and the Problem Solving Sessions; [7] Literature Relevant to Problem Solving Strategies for the Classroom and the Problem Solving Sessions; [8] Literature Relevant to the Problem Solving Beliefs and Attitudes of the Students in the Classroom and the Problem Solving Sessions.

The literature reviewed in this chapter informs the collection, analysis and discussion of the data in both the classroom and the problem solving sessions and, as such, will be referred to in the relevant chapters pertaining to the presentation and discussion of the two sets of data.

Literature Relevant to Change in the Teaching and Learning of

Mathematics and Mathematical Problem Solving

This study explores changes in the teaching and learning of mathematics and mathematical problem solving, specifically addressing those concerns that accompany changes from a traditional perspective where students learn mathematics at a rote level, without understanding how to apply what they know, to problem solving situations [NCTM, 1980; NCTM, 1983; McKnight et al, 1987; NCTM, 1989; Crosswhite, 1990; NRC, 1990; Romberg, 1990; NCTM, 1991; Kamii, 1994]. The following literature has informed this study by providing information on why such changes are necessary and how the suggested changes affect the teaching and learning of mathematics and mathematical problem solving.

The following bodies of research have set forth strong recommendations for change in the present mathematics curriculum: *An Agenda for Action: Recommendations for School Mathematics in the 1980's* [NCTM, 1980]; *A Nation at Risk* [NCEE, 1983]; *The Underachieving Curriculum: Assessing US School Mathematics from an International Perspective* [McKnight, Crosswhite, Dossey, Kifer, Swafford, Travers & Cooney, 1987]; the *Standards* [NCTM, 1989]; *Everyone Counts* [NRC, 1989], *Reshaping School Mathematics* [NRC, 1990]; *Professional Standards for Teaching Mathematics* [NCTM, 1991]; *Assessment Standards for School Mathematics* [1995]. These documents indicate that parents, administrators, researchers, teachers and other interested people agree that problem solving should be the highest priority in the mathematics curriculum. Large scale reforms to mathematics curriculum and its teaching to include problem solving opportunities are recommended [Stonecipher, 1986; NCTM, 1989; Becker, 1989; Dossey, 1989; Romberg, 1990]; *Priorities in School Mathematics* [1981].

Why is there such a critical need for enhancement of problem-solving in the curriculum? Becker [1990] and NCEE [1983] indicate that knowledge, learning, information and problem solving have become the new raw materials of the world today. Business leaders complain that millions of dollars are spent on costly remedial education and training in mathematics [NCTM, 1980; NCEE, 1983; NCTM, 1989]. Businesses no longer seek workers with arithmetic skills traditionally associated with shopkeepers [NCTM, 1980, NCEE, 1983; NCTM, 1989].

However, the need for a larger focus on problem solving in the curriculum derives from the belief that all students are entitled to the best possible education that enables them to become successful members of society. This focus should include the knowledge, skills and attitudes that allow students to explore various alternatives. Students need successful experiences in problem solving so that they will have the opportunity to live interesting and productive lives [NCTM, 1989].

The *Standards* [1989] emphasize a "comprehensive and rich approach to problem solving in a classroom climate that encourages and supports problem solving efforts. Classrooms with a problem solving orientation are permeated by thought-provoking questions, speculations, investigations and explorations" [p. 23]. The knowledge of a variety of techniques to approach problems, the ability to set up problems with appropriate operations, understanding of the underlying mathematical features of a problem, the ability to work with others on problems, the ability to apply mathematical ideas to complex problems, preparation for open problem situations and belief in the usefulness of mathematics are all mathematical expectations for students [NCTM, 1989].

Mathematics has become a critical prerequisite for employment and

full participation in society. A student must be prepared to become a flexible member of the workforce, capable of lifelong learning. This implies that school mathematics must emphasize dynamic approaches that encourage excitement in mathematics. Mathematics educators must encourage students to explore, to create, to accommodate to changing conditions and to actively create new knowledge [Suydam, 1982; Thompson & Rathmell, 1988; NCTM, 1989; Becker, 1989].

This study attempts to address some of the changes arising from the recommendations discussed above, specifically changes to the curriculum that place emphasis on problem solving.

Literature Relevant to the Classroom Mathematical Inquiry

For eight months, observation and documentation was carried on in a selected elementary classroom described by the teacher as inquiry oriented. The purpose of this classroom observation was to explore and document the nature of this mathematical inquiry in the classroom. The following literature provided the necessary information on what constitutes an inquiry-oriented approach to mathematics and mathematical problem solving

Literature on inquiry-oriented teaching and learning in mathematics and mathematical problem solving that informed the nature of mathematical inquiry in the selected classroom includes that of Yoshikawa [1989]; Garofalo [1989]; Becker [1990]; Becker and Shimada [1993] and Novak [1977] who describe an inquiry approach as one in which students use their previously learned knowledge, skills, beliefs, attitudes and thought processes to solve problems and to learn something new in the process.

Tougaw [1993], in his study of the effects of "open instruction" on

the problem solving practices of students, sees this "open" approach as similar to an inquiry-oriented approach to instruction. An "open" inquiry approach includes conjecture, exploration, discovery, discussion, verification and generalization which draws on students' own previously learned attitudes, skills, knowledge and thought processes in solving mathematical problems. Examination of the behaviors, strategies and methods of solution that subjects use during mathematical problem solving is essential to Tougaw's study. For the purposes of this study, Tougaw's [1993] description of his "open" approach was informative because his description was similar to and supportive of the teacher's description of her inquiry-oriented approach. Tougaw's [1993] description of an inquiry-oriented approach, the "vision" of the NCTM *Standards* [1989] and the teacher's description suggested three topics for the exploration and documentation of the mathematical inquiry in the selected classroom.

Garofalo [1993] informed both the classroom data and the problem solving data by describing students learning mathematics and mathematical problem solving in an inquiry approach as "meaning oriented" problem solvers who approach problems by trying to form meaningful interpretations of the conditions and quantities in the problems, construct meaningful relationships between the quantities and then use these understandings to develop meaningful plans of action. According to Garofalo [1993], these "meaning oriented" problem solvers read problems to find the meaning and to comprehend better. After working for a short while, they go back to problem statements to think about the questions or to see if they make sense or if the method is correct.

Other research that informed the study included that of Silver & Mamona [1989] who suggest that, through experiences in solving open-

ended problems, students can develop powerful problem solving abilities. According to these researchers, the use of an inquiry-oriented approach involves a diversity and variety of problem solving activities including providing students with an opportunity to solve unfamiliar problems. This approach seeks to optimize the learning process by inviting the student to become personally involved and to actively communicate, discuss and compare solutions, answers and approaches [Becker, 1992; Becker, 1993]. Such problems encourage students to improve their critical-thinking skills and higher-order thinking, discover mathematical concepts and problem solving strategies, solve real-life problems and check their results [Becker, 1992; Becker & Shimada, 1993; Becker, 1993].

The following bodies of literature also informed the study by providing additional information on the nature of the mathematical tasks presented to students in an inquiry-oriented classroom. Essential to an inquiry-oriented approach is the development or selection of problems that are intended to provide an opportunity for any student, regardless of ability, to find at least one solution and participate in the discussions. Creating more opportunities for students of all abilities to actively participate in the classroom problem solving process leads to increased confidence in their ability to do mathematics [Nagasaki & Becker, 1993; Becker, 1993]. The classroom activities implicit in an inquiry-oriented approach help students to improve their critical-thinking skills, develop problem solving strategies, solve real-life problems and check their results. Students observing other students' discoveries or methods, comparing and evaluating students' different ideas and modifying their own ideas accordingly are important aspects of the inquiry approach.

This approach to mathematics is described by the NCTM [1989] as

one that emphasizes students' thinking strategies. The *Standards* [1989] articulate the following goals for K-12 students: Through such an approach, students learn to value mathematics, become confident in their abilities to do mathematics, become mathematical problem solvers, learn to communicate mathematically and learn to reason mathematically [p. 5]. To develop these five abilities, the NCTM [1989] suggests that students need to work on problems that may take hours, days and even weeks to solve. These problems could be solved independently, require work in small groups or involve an entire class working cooperatively. Many problems should be open-ended to encourage multiple solutions. In such an inquiry-oriented approach to mathematics, problems may offer many approaches to finding a unique answer. The multiplicity of a possible number of solutions and varied techniques for finding those solutions create an environment in which students are encouraged to use their own previously learned attitudes, skills and ways of thinking [Becker & Shimada, 1993]. Such an approach causes students to focus on and develop different possible techniques and strategies for reaching solutions to problems. Finding several different solutions to a problem invites students to experience the productive side of mathematics. The activity of solving open-ended problems in this inquiry-oriented approach invites students to explore, discover, analyze, verify, generalize, discuss mathematical ideas and view mathematics as an exciting discipline [Yoshikawa, 1989; Garofalo, 1989; Becker, 1990, Becker, 1992; Becker & Shimada, 1993; Becker, 1993]

Other literature on inquiry-oriented teaching and learning in mathematics includes that of Seely [1988] which explores inquiry-oriented classrooms and the relationship between such an approach and the social construction of knowledge. Seely et al [1988] state that knowledge is not

independent, but is fundamentally "situated" being in part a product of the activity, context, and culture in which it is developed. Seely's [1988] work informed the study because of the discussion on “situated learning”. Throughout the classroom data, there was a significant body of information which addressed the support of the broader context of the school, including parents and administrators. Seely [1998] refers to this context as “situated learning” which calls for learning and teaching methods that take into account the support of the school community for the teaching and learning of mathematics and mathematical problem solving in the selected classroom. This is in contrast to traditional methods which overlook the central contribution made by activities, context and culture of schools to what is learned here.

The following bodies of literature address the changes that occur in making problem solving a priority. Souviney [1981] suggests that the main responsibility of schools is to help children become effective problem solvers, thus helping to prepare students to become functioning members of society. Teachers are being encouraged to make problem solving a priority for the mathematics curriculum [Greenes & Schulman, 1982; Suydam, 1982; Thompson & Ratmell, 1988].

In contrast to the inquiry-oriented approach, students in the traditional classroom view mathematics as a collection of exercises solved quickly using learned facts, rules and procedures [Frank 1988]. According to Frank [1988], when faced with a problem solving task, students in the traditional classroom adopt one of three approaches: The first is not to accept the problems as true mathematics and refuse to attempt or to make a weak attempt at the solution. The second is to attempt the problem as if it was a textbook exercise and seek a rule or procedure leading to the quick

answer. The third is to try to employ a general problem-strategy for a short period of time. Frank [1988], found that these approaches appear to contradict the problem solving process. Frank's work [1998] was relevant to the study and informed the discussions on both the classroom data and the problem solving data in relation to the effect of student's beliefs on their problem solving practices.

One disadvantage of an inquiry-oriented approach to mathematics and mathematical problem solving is that teachers often struggle with the lack of a proper pedagogy to teach problem solving. The idea that teachers find it difficult to successfully pose problems and develop mathematically meaningful problem situations [Nagasaki & Hashimoto, 1984] is an important factor in this study.

Literature Relevant to the Classroom Organization for Problem Solving

In the selected classroom, the teacher organized her students into problem solving groups. The following literature informed the study on the relationship of group work in mathematical problem solving to mathematical problem solving competency.

Noddings [1985] and Schrader [1985] have suggested that the use of small cooperative groups similar to the problem solving groups in the selected classroom result in significant growth in mathematical problem solving competence. Research by Lochhead [1979], Whimby [1980], Schoenfeld [1983], Bellanca [1984] and Hoomes [1984] demonstrates that such small-group work facilitates growth in problem-solving skills, allows students to observe the behaviors of others and allows participation in group discussions. Students observing other students' discoveries or

methods, comparing and evaluating students' different ideas and modifying their own ideas accordingly are important aspects of an inquiry-oriented approach [Becker, 1992; Becker & Shimada, 1993; Becker, 1993]. Irons & Irons [1989] suggest that students' knowledge and excitement about mathematics increases if teachers provide experiences that allow discussion, an extension of known strategies and the development of new techniques.

McDonald's research [1990] about the role of cooperative learning in mathematics suggests that classmates in a mathematics class expect to help each other and that the amount and type of this interaction is largely determined by the students' personalities, actual mathematical ability and self-perceived mathematical ability. McDonald's research was particularly relevant to the second set of data in relation to the students' non-mathematical practices.

Artz [1996] states that small group settings appear to provide a natural environment in which increased dialogue and communication about mathematics can occur among students. However, according to Artz [1996], for positive outcomes to occur in small groups, activities must be structured to maximize the chances that students will engage in questioning, elaboration, explanation and other verbalizations in which they can express their ideas and through which the group members can give and receive feedback. Artz [1996] comments that, without structure, students may just share answers, do one another's work or they may help one another, with a multitude of possible communication patterns in between. This research informed the study in relation to how the classroom teacher structured her groups and how her structuring of groups affected the students' problem solving practices.

In the selected classroom, a multi-age organizational strategy was

observed. Opuni & Koonce, [1992] suggest such an arrangement for classrooms as one whereby children of various ages, abilities and interests are placed together in a learning environment. The research of Opuni and Koonce [1992] provided the rationale regarding this way of working with children, providing "the opportunities for inter-age challenge, mentoring, support, friendships, leadership, peer and cross-tutoring" [p. 7].

Literature Relevant to the Classroom Teacher's Role in Problem Solving

The teacher of the classroom selected for this study described her approach to the teaching and learning of mathematics as an inquiry-oriented approach. The teacher's role is an important component of such an approach. The following literature provided the necessary information on the teacher's role and also on the multiple layers of teaching practices demonstrated by the teacher in the selected classroom in relation to her inquiry-oriented approach.

The research of Goos [1996] presents a model of the teacher's interactions with the students as the teacher works towards creating a culture of mathematical sense-making. The results indicate four aspects of the teacher's role that are particularly important in establishing a classroom community of practice: [1] modeling mathematical thinking; [2] cognitive and social scaffolding; [3] encouraging individual reflection, self-monitoring and checking and [4] introducing tools for mathematical communication.

Examination of the literature concerning problem solving over the last twenty-five years provides a number of insights for teachers: [1] be clear about your goals; [2] understand that problem solving is a highly

complex process; [3] be prepared to find that problem solving performance is difficult to improve; [4] encourage students to be active participants in their problem solving and [5] provide a safe, congenial environment for students to practice problem solving [Polya, 1981; Silver, 1982; Good, Grouws & Ebmeier, 1983; Lester, 1985; Kilpatrick, 1985; Becker & Shimada, 1993].

The *Professional Standards* [1991] describe the discourse of the classroom [activity and discussion] as central to students mathematical learning. The teacher's role is to pose problems and tasks that provoke and challenge students to think, ask students to classify and justify their ideas, decide what to pursue in depth from among the ideas that students bring up during a discussion and decide what information to provide. The role of the teacher is also to let students struggle with a difficulty and to monitor students' participation in discussions [NCTM, 1989; NCTM, 1991].

Both the *Standards* [1989] and the *Professional Standards* [1991], state that teachers of mathematics should have the skills to pose tasks that are based on sound knowledge of mathematics, knowledge of how students learn mathematics, knowledge of students' understandings, interests and experiences and knowledge of the necessary pedagogy. Activities should engage students' intellects, foster students' understandings of concepts and procedures, help students to make connections and promote problem formulation, problem solving, reasoning and communication.

The research of Stigler, Gonzales, Kanwanka, Knoll & Serrano [1999], as published in *The Third International Mathematics and Science Study Videotape Classroom Study* and the research of Ball [1996], Battista [1994] and Schifter [1996] informed the final discussion of the study on teachers' understandings of the NCTM reforms This research provided

insights into teachers' struggles between their interpretations of the NCTM "vision" and their own, embedded beliefs about teaching and learning mathematics and mathematical problem solving.

Literature Relevant to the Routine Problems Presented in the Problem Solving Sessions

In this study, four students from the selected classroom were chosen to work on seventeen sets of routine traditional problems. The literature presented in the following section informed the study by providing definitions of routine problems and the effect of routine problems on the problem solving practices of the students. The literature also informed that part of the study on the nature of mathematical inquiry in the selected classroom, specifically, on the nature of the problems presented to students.

According to Becker [1983, 1990], and Stonecipher [1986] many mathematical problems that appear in textbooks and classroom activities are not problems at all; rather, they are routine exercises with a new name. These problems, similar to the problems presented to the four selected students in this study, are usually prefaced by examples and explanations that fully describe the correct solution technique. Schoenfeld [1983] suggests that, after being so well prepared by authors and teachers, students come to think that all mathematics is known, that it is a finished system and must be rehearsed until it is learned. Such exercises deprive students of the process of discovery, of the satisfaction that can be derived from this process and from any sense of what understanding mathematics means. Polya [1963] criticized typical textbook problems included in most school texts as short-ranged, routine examples that merely illustrate the application of one rule.

Bottge & Hasselbring [1993] describe routine problems as problems that appear to connect with real life experiences but, in reality, describe situations textually rather that contextually and seem artificially geared to specific number operations and single correct answers. The research of Bottge & Hasselbring [1993] suggests that students do not associate these routine, traditional, word problems with their own experiences because these problems are perceived as making only artificial, "real-world" connections. This concept of "real world" connections [Bottge & Hasselbring, 1993] inform the description of the problems in both the classroom and the problem solving data.

Lochhead [1981], Heller & Hungate [1985] suggest that routine problems, such as the problems presented to the students in this study and found in most textbooks, place students in the role of copiers where they are less active because they do not believe that there is any more for them to do. Krutetski [1976] observes that students who over rely on number considerations put much more of their time and energy into activities related to calculation. These efforts include checking computations excessively. Skemp [1987] addresses this type of mathematical problem solving as "rules without reasons", that is, using mathematical rules without understanding why or where these rules came from. Duncan [1985] presupposed that mathematical structures may be cued by elements independent of an understanding of the problem context. Duncan also suggests that the nature of the routine problems themselves with their emphasis on specific operations and concrete answers, appeared to direct the students into problem solving behaviors that also focus on numbers, calculations, key words and performance. Overson [1989] refers to students' attitudes towards solving routine problems as getting mathematics

"over with". Overson's work [1989] informed the discussion of getting mathematics "over with" primarily in relation to the problem solving data.

According to the *Standards* [1989], students need many and varied experiences with problems for which there is no routine or obvious answers. Researchers and theorists have defined mathematical problems in a number of different ways. Polya [1981] states that a problem is defined as a situation that confronts an individual or group of individuals and that requires a resolution and for which an individual has no apparent or obvious means to obtain a solution. Polya intended his theory of problem solving to go beyond the classroom into real-life problem solving. He states:

> In mathematics, know-how is much more important than mere possession of information. What is know-how in mathematics? The ability to solve problems, not merely routine problems but problems requiring some degree of independence, judgment, originality and creativity. Therefore, the first and foremost duty of teaching mathematics is to emphasize problem solving [p. 12].

Lester [1980] defines a problem as a situation in which an individual or a group was called upon to perform a task for which there was no obvious algorithm which could be used in finding a solution. Similarly, Kantowski [1977] defines a problem as a situation where an individual encounters a question he cannot answer or a situation he is unable to resolve using the knowledge immediately available to him. Kantowski [1977] states that an individual "must then think of a way to use the information at his disposal to arrive at the goal, the solution of the problem" [p. 163]. Shimada [1977] states that a good problem is one which causes students to develop higher-order thinking skills by focusing their efforts on different possible methods, techniques and strategies for getting

answers to problems.

Dewey [1916], an advocate of school reform, defined a problem as anything that gives rise to doubt and uncertainty. A good problem, says Dewey, must be of importance to the culture/society and relevant to the students [in Stonecipher, 1986].

Literature Relevant to Defining Problem Solving for the Classroom and the Problem Solving Sessions

This study addresses problem-solving practices in both the classroom and in the problem solving sessions. The literature in this section informed the definition of problem solving for the study in both the classroom data and the problem solving data.

A general definition of problem solving for this study is that it is a process that involves reorganization of stored information and accumulation of new ideas into the cognitive structure of the problem solver to reach a specified goal [Novak, 1977]. Problem solving has also been defined as what you do when you do not know what to do [Frank, 1988].

There is a general consensus concerning what actually constitutes problem solving and what the variables are that influence successful problem solving [Lester, 1980]. Many discussions of mathematical problem solving include some form of the teachings of Polya [1945, 1954, 1963, 1981]. For over sixty years, Polya taught and wrote about mathematical problem solving. His ideas and teachings focused on problem solving as a way of teaching and learning mathematics [Alexanderson, 1987]. Polya defined problem solving thus:

> To solve a problem is to find a way where no way is known, off-hand, to find a way out of a difficulty, to find a way around an obstacle, to attain a desired end that is not immediately attainable by appropriate means [from Krulik, 1980, p.1].

In his approach, Polya [1945, 1954, 1963, 1981] searched for patterns of productive thinking that enabled him to be successful at mathematical problem solving. Believing that he might discover a way to allow others to experience success in problem solving, he embarked on the study of problem solving strategies which have been widely reported for many years and which he himself reported. Polya's [1945] conception of mathematics problem solving was that of a four-phase heuristic process: understanding, planning, carrying out the plan and looking back.

Many of the approaches to teaching mathematical problem-solving that have been advocated in recent years have been roughly classified into five categories [Kilpatrick, 1985]. These categories informed the study in both the classroom problem solving and the problem-solving sessions, where models were identified from these five categories and applied to student problem solving strategies in order to describe the problem solving strategies in both sets of data. These five categories are: [1] osmosis - a process involving immersion of the students in an environment of problems with the assumption that problem solving techniques will be absorbed while solving these problems [Covington & Crutchfield, 1985]; [2]; memorization - a technique where an algorithm was developed that will handle a class of problems and the students were trained to identify and implement the appropriate algorithm [Dahmus, 1970]; [3] imitation - a process where the students were trained to model a master problem solver [Bloom & Broder, 1950; Covington & Crutchfield, 1985]; [4] cooperation

- an environment that used group problem solving sessions as a vehicle for research [Lesh & Akerstrom, 1982; Schoenfeld, 1982; Dees, 1983]; and [5] reflection - a process proposed by Dewey [1916] and Piaget [1928] utilizing metacognition where students learn by doing and thinking about why they performed the way they did [Papert, 1975; Silver, 1982; Schoenfeld, 1983].

Successful problem solving instruction needs to transform problems from the realm of school tasks to be completed as quickly as possible to intellectual challenges that are viewed as worthwhile endeavors by the students [Kilpatrick, 1985]. Kilpatrick [1985] commented that information gathered from many sources suggests that the best answer to how problem solving is learned is "slowly and with difficulty". Kilpatrick [1985] also states that research on problem solving includes the convergence of research efforts of cognitive scientists and mathematics education researchers on problem solving, the emergence of mathematics teachers as partners in research, and efforts to move the research out of the laboratory and into the classroom [Kilpatrick, 1985]. As Kilpatrick [1985] suggests, this study took place primarily in a classroom and involved the teacher in the research.

To further highlight the differences between routine and non-routine problems an example from the *Professional Standards for Teaching Mathematics* [1991, p. 28] is now presented. In this example a teacher is thinking about how to help her students learn about perimeter and area. The teacher realizes that learning about perimeter and area entails developing concepts, procedures and skills. Students need to understand that the perimeter is the distance around and the area is the amount of space inside the region and that length and area are two fundamentally different

kinds of measure. They need to realize that perimeter and area are not directly related - that, for instance, two figures can have the same perimeter but different areas. Students also need to be able to figure out the perimeter and the area of a given region. At the same time, they should relate these to other measures with which they are familiar, such as measures of volume and weight.

The teacher examines two tasks designed to help upper elementary grade students learn about perimeter and area. She wants to compare what each has to offer.

The first task states : Find the area and perimeter of each rectangle [included is a diagram of a rectangle with a length of 20 cm. and a width of 12 cm. and a square with each side measuring 16 cm.].

The second task states : Suppose you had 64 meters of fence with which you were going to build a pen for your large dog, Bones. What are some different pens you could make if you use all the fencing? What is the pen with the least play space? What is the biggest pen you can make - the one that allows Bones the most play space? Which would be the best for running?.

The teacher now chooses which task she wants for her students:

Task One requires little more than remembering what "perimeter" and "area" refer to and the formulae for calculating each. Nothing about this task requires students to ponder the relationship between perimeter and area. The task is not likely to engage students intellectually; it does not entail reasoning or problem solving.

Task Two can engage students intellectually because it challenges them to search for something. Although accessible to even young students, the problem is not immediately solvable neither is it clear how best to

approach it. A question that students confront as they work on the problem is how to determine that they have indeed found the largest or the smallest play space. Being able to justify an answer and to show that a problem is solved are critical components of mathematical reasoning and problem solving. The problem yields to a variety of tools - drawings on graph paper, constructions with rulers or compasses, tables, calculators, and lets students develop their understandings of the concept of area and its relationship to perimeter. They can investigate the patterns that emerge in the dimensions and the relationship between those dimensions and the area. This problem may also prompt the question of what "largest" or "smallest" "most" or "least" means, setting the stage with making connections in other measurement contexts.

These samples of two problem tasks related to the same mathematical concept illustrates how this study looks at both approaches to problem solving in the classroom; the routine problems , as presented in Task One above and non-routine [inquiry-oriented] problems , as presented in Task Two above. Both types of problems are typical of what was seen in the classroom selected for this study.

Literature Relevant to Problem-Solving Strategies for the Classroom and the Problem Solving Sessions

This study addresses the practices of problem solvers. The following literature informed the study by identifying the characteristics demonstrated by good problem solvers. These characteristics were used in order to establish the levels of problem solving competency demonstrated by the students both in the classroom and in the problem solving sessions.

Proudfit's [1980] research on characteristics which successful

problem solvers display which non-successful problem solvers do not display suggests nine behaviors which were found to accompany successful problem solving. These behaviors are: focusing on relevant information, focusing on the questions being asked, explaining the reason that a strategy seemed appropriate for a particular problem, explaining the implementation of the strategy, evaluating the strategy, evaluating the solution in light of the problem conditions, drawing a diagram, making the problem more concrete, making a model of the total situation.

Duncan [1985] identifies and analyzes problem solving strategies and the settings in which these strategies could be optimized. The findings indicate that intermediate aged children, when solving a variety of routine and non-routine problems in small groups, display general patterns of behavior. These include: [1] the manner in which the groups approach and effectively isolate the contextual elements of a problem; [2] the propensity of the groups to change the mode in which a problem is represented by utilizing manipulatives, diagrams, tables and other physical displays and [3] the manner in which groups monitor the course of problem solving and reach consensus on solution proposals.

Successful problem solvers are characterized by the ability to comprehend mathematical concepts and terms, determine likenesses, differences and analogies; identify critical elements and select correct procedures, eliminate irrelevant information, analyze and estimate answers more accurately and completely and analyze pertinent information more quickly, accurately and with confidence. Good problem solvers focus on important structural features of the problem, whereas poor problem solvers focus on surface details [Stonecipher, 1986].

Driscoll [1983] indicates that good problem solvers demonstrate the

ability to evaluate and select alternative solutions, use estimation and approximation strategies and check for reasonableness of answers. Kagan [1964], Robinson [1973], Krutetski [1976] and Kantowski [1977] report that good problem solvers tend to spend more time attempting to solve unfamiliar problems, to be more persistent on difficult tasks and be more systematic in their approach than are poor problem solvers. Lochhead [1981], Schoenfeld [1983], Garofalo & Lester [1985] suggest that good problem solvers have well-developed skills for representing mathematics problems and tend to perform qualitative analyses of problems before doing any computation.

Literature Relevant to the Problem-Solving Beliefs and Attitudes of the Students in the Classroom and the Problem Solving Sessions

The problem-solving beliefs of students and their parents were an integral and important part of the discussion for this study. The literature in this section informed the classroom data, the problem-solving data and the relationship between the two sets of data by presenting the nature of these problem solving beliefs and their effect on students' mathematical problem solving. The information provided by the literature formed an important part of the conclusions, final discussions and implications of the study.

There is a general agreement that affective characteristics such as motivation, interest, self-confidence, anxiety and perseverance are essential to problem solving [Stonecipher, 1986]. Debellis [1996] describes and explores four children in the context of mathematical problem solving. The findings suggest a major, complex interplay of the affective domain

with children's mathematical thinking. Artz, Armour & Thomas [1992] and Curcio & Artz [1995] found that a continuous interplay of cognitive [the "doing" of mathematics] and metacognitive [the choosing, planning, monitoring and regulating of mathematics] behaviors is necessary for successful problem solving and maximum communication among students. Goos [1996] states that students' perceived status in the group inhibit effective problem solving.

Research studies recognize that the context in which mathematics is taught and learned [Cobb, 1987, 1992] and the beliefs about the nature of mathematics and mathematical tasks [Schoenfeld, 1987] have profound effects on mathematical performance. This study explores the idea that the nature of the classroom environment in which mathematics is taught strongly influences how students view the subject of mathematics, the way mathematics should be done and what students perceive as the appropriate response to mathematics problems [Garofalo, 1989]. Walkerdine [1988] has written that behaviors in learners are produced because that is what the educational task is set up to do.

The way in which the mathematical tasks in this study illustrate a particular pedagogic and discursive style seem to be particularly important. Cobb [1984] states that students' beliefs are developed from the perceived expectations of the teacher and these beliefs often do not reflect healthy and realistic views of mathematics. Studies carried out by Frank [1988] and Garofalo [1989] reveal the following student beliefs and attitudes about mathematics: [1] all mathematical problems are solved by the application of mechanical step-by-step procedures; [2] common sense responses and reasoning are not relevant to in-school exercises; [3] the teacher and the text-took are the only sources of mathematical truth and correctness; and

[4] there is no need for students to attempt to evaluate their answer. Carpenter et al [1988] report that students regard mathematics as consisting of memorizing and solving problems using rules.

Garofalo [1993] suggests that students' problem-solving styles are influenced by the personal mathematical goals of the problem solver. For the students in this study, Garofalo's [1993] work is particularly important. His research on a "number-oriented" style of problem solving informed the discussion of the students problem solving styles. A "number-oriented" style of problem solving, according to Garofalo [1993], is preferred by students whose only goals are to get a sufficient number of problems correct in order to satisfy their teachers and parents and to avoid failure.

Student beliefs about the nature of the mathematics are a direct product of the environment in which mathematics is taught and learned [Frank, 1988]. Garofalo [1989] hypothesized that students beliefs and attitudes directly affect mathematical problem solving strategies.

Echols [1981] found that the attitude of parents is the most important predictor for students' attitudes towards mathematics. Schneider [1984], Husen & Postlethwaite [1985] reviewed several studies on the attitudes of parents and their effect on the attitudes of their children in mathematical problem solving. They found agreement with the studies of Echols [1981]. The literature on the attitude of parents towards mathematics informed the discussions about the students' engaging in and completing mathematical tasks.

Researchers such as Fey [1979]; Confrey & Lanire [1980]; Buerk [1982]; Schoenfeld [1983], Silver [1982]; Carpenter et al [1983, 1998]; Wheatley [1984]; Confrey, [1984, 1995]; Cobb [1984, 1992] and Frank [1988], concur that curriculum innovations are not sufficient to improve

problem-solving skills in students. Confrey [1984, 1995] states that the successful implementation of problem solving instructional approaches requires changes in students' beliefs and attitudes towards mathematics. Students do not become better problem solvers unless they change their beliefs about mathematics.

The findings of the researchers above reveal the following list of beliefs that students hold about mathematics and that were of importance for this study: [1] mathematics is computation and doing mathematics means memorization and following rules; [2] mathematics problems are quickly solvable in just a few steps; [3] the goal of doing mathematics is to obtain the "right answers"; [4] the role of the students is to receive the mathematics and demonstrate this reception of information on the next test and [5] the role of the teacher is to transmit the mathematical knowledge and to verify that students have received this knowledge [Frank, 1988].

Summary

This study addresses the large-scale reforms in mathematics education which have called for an improvement and revision of the mathematics curriculum and its teaching to include problem solving opportunities.

The inquiry-oriented approach to the teaching and learning of mathematics and mathematical problem solving in the selected classroom embodies many of the ideas presented in this review of research literature. Attributes of the inquiry-oriented approach to teaching and learning in mathematics and mathematical problem solving, as presented in the literature, include the use of carefully selected and prepared open-ended problems; the teacher as facilitator and orchestrat0r of classroom lessons;

and individual, small-group and whole-class discussions. Mathematical problem solving has been described in this study as an active process involving thought-provoking problems, exploration, discovery, discussions, verifications, speculations, data collection, investigations, generalizations and higher-order thinking processes. Good, thought-provoking problems must originate from the learning objectives for the class, be non-routine, have solutions that are not readily apparent, be readily understandable by the student, be self-motivating, suggest new problems, encourage extensions and integrate various subject areas into the solution.

Routine, traditional mathematical problems, such as those found in typical school mathematics textbooks and as presented to the students in the problem solving sessions, are not problems at all but routine exercises. Such exercises cause students to be deprived of the satisfaction that can be derived from this process and from any sense of what understanding mathematics means. The *Standards* [1989] suggest that students need many and varied experiences with problems for which there are no routine or obvious answers.

The difference between non-routine and routine problems are that non-routine problems do not have an obvious algorithm that, when applied, would certainly lead to a solution [Polya, 1945; Kantowski, 1977; Schoenfeld, 1983; Becker, 1983; Yoshikawa, 1989; Becker & Shimada, 1993]. Typical mathematics textbooks yield very few genuine problem-solving experiences for students beyond the routine, conventional exercises that require only one-step problem solving techniques [Becker, 1990]. A good problem should offer a change from routine problems and offer an opportunity for students to engage in inquiry-oriented thinking, lead to more effective communication among students and possibly lead to many

solutions and forms of the answer [Fehr & Swanson, 1988].

A number of observations are relevant to this study. Technology has increased the need for an understanding of mathematics and a strong conceptual base in mathematics is important for students to function successfully in a constantly changing society. Knowledge of computation and a ready understanding of procedures are no longer the only skills needed by students today. Students are now required to use higher-level thinking skills and to determine alternate solutions to problems [Lindquist, 1989].

Inquiry-oriented teaching and learning in mathematics and mathematical problem solving has the power to expand the narrow view that mathematics is a tool for solving problems to the broader conception that mathematics is a way of thinking about and organizing one's experiences. Learning should become self-generated to allow students to develop concepts, thinking skills and strategies and determine how to manage and regulate the application of this new knowledge [Schroeder & Lester, 1989].

The literature reviewed in this chapter on the current changes in the teaching and learning of mathematics and mathematical problem solving provides a context for the study of the importance of problem solving in modern society and the concerns that often accompany such changes. The research on inquiry-oriented teaching and learning in mathematics and mathematical problem solving provides a vision of what classroom mathematics and mathematical problem solving can be and the tension between this vision and what often occurs in mathematics classrooms.

The literature informs the description of mathematical inquiry in the selected classroom, the classroom organization for problem solving and the

classroom teacher's role in problem solving. The literature informs the nature of the routine problems presented in the problem solving sessions. In relation to both the classroom data and the problem-solving data, the literature informs the definition of problem solving for the classroom and the problem solving sessions, problem solving strategies for both the classroom and the problem solving sessions and the problem solving beliefs and attitudes of the students in the classroom.

The literature presented in this chapter also informs the discussion on the relationship between the two sets of data; that is between the teaching practices of the teacher and the problem solving practices of the students. This discussion is crucial in addressing the purpose of this study; namely, the concerns in relation to the tension between the NCTM's major focus on problem solving and teachers' efforts in developing mathematically meaningful problem situations [Nagasaki & Hashimoto, 1984].

CHAPTER III

METHODOLOGY

This chapter discusses the framing of the study within a qualitative research methodology. The chapter includes a general overview of qualitative research pertaining to this study as well as details of the data collections and analyses. Also included is a discussion of the trustworthiness and ethical considerations of the study.

The guiding question of the research, "What practices are elicited when four Grade III/IV students, learning mathematics in what is described by their teacher as an inquiry-oriented approach, are presented with routine, traditional mathematical problems?" required two data collections and analyses. The two data collections were necessary in order to address: [1] the exploration and documentation of the nature of mathematical inquiry in an elementary classroom; [2] the problem solving practices of four students from this classroom when they are presented with routine, traditional problems. The first data collection and resulting analysis took place in the selected classroom. The second set of data were collected and analyzed from six problem-solving sessions.

A Qualitative Framework

In order to establish a research methodology for a study, it is necessary to consider the nature of the research [Yin, 1984] and the kind of response the question requires [Weber, 1986]. This study is qualitative in nature because it is framed around a question that suggests an exploratory approach. In addition, the nature of the study is qualitative because the

anticipated response is a deeper understanding of the teaching and learning of mathematics and mathematical problem solving referred to by Marshall & Rossman [1995] as the social phenomena.

This research explores mathematical teaching and learning and mathematical problem solving and attempts to interpret the phenomena in terms of the meanings brought to them by the people involved in this study. According to Gall, Borg & Gall [1996], such qualitative studies play an exploratory role in order to discover themes and relationships and are defined as individuals constructing social reality in the form of meanings and interpretations. The study values setting and context with the classroom as a source of data and the researcher being the key instrument of data collection and analysis.

In this study, the descriptive nature of the data is crucial in understanding the teaching and learning of mathematics and mathematical problem solving. Description is used to document the mathematical teaching and learning in the classroom and the problem solving practices of the selected students as they worked on mathematical problems. This study renders primacy to the process of mathematical teaching and learning and mathematical problem solving. The flexible study design allows for inductive data analysis during and upon completion of the data collections [Bogdan & Biklen, 1992].

This study took place primarily in a school and, therefore, took the form of a field study, described by Spradley [1980] as "learning from people" [p. 3]. Data for the study were collected through a participant observer approach [Spradley, 1980]. Observations were recorded in field notes and a dialogue journal. Data were also collected through audio tapes and artifacts. A summary of the qualitative strategies, data collection and

analysis is presented in Figure 1.

Figure 1: Matching the Research Question with the Qualitative Strategies, Data Collection and Analysis

Research Question	Nature of the Study	Qualitative Research Strategy	Strategies for Data Collection and Analysis
What practices are elicited when four Grade III/IV students learning mathematics in, what is described by their teacher as, an inquiry-oriented approach, are presented with routine, traditional mathematical problems?	• Values context • Process is seen as crucial • Recognizes the importance of descriptive data • Flexible design • Inductive data analysis	• Field Study • Participant Observation in the classroom •Participant Observation in the problem solving sessions in conjunction with audio taping	• Field notes • Dialogue Journal • Audio tapes • Artifacts • Data is analyzed inductively throughout and on completion of the data collections.

Overview of the Data Collections

This study took place over a 10 month period. Two sets of data were collected.

Data were first collected in a Grade III/IV classroom in order to explore and document the nature of teaching and learning of mathematics and mathematical problem solving in the classroom. The data were recorded in field notes and a dialogue journal with the classroom teacher. Artifacts, such as copies of students' report cards, notices to parents, copies of posted notices from the classroom and copies of students' work from

their mathematics journals, were also collected.

Following the collection of the first set of data, a second set of data were gathered. These data consisted of transcriptions of the audio taped talk of four students, chosen from the selected classroom, as they worked on routine, traditional mathematical problems which were presented to them in a setting apart from the classroom and the teacher. Field notes were also collected in this second set of data.

Each of the two sets of data collections are discussed below.

Data Collection #1: The Classroom Data

This first set of data for this study were collected in a selected Grade III/IV classroom for two days a week, usually Wednesday and Thursday, for eight months from mid-September, 1994 to April, 1995. This amounted to approximately two hundred hours in the classroom. The purpose of this first set of data was to explore and document the inquiry-oriented approach which the teacher indicated was used in her teaching, and the learning of mathematics and mathematical problem solving. A description of the school in which the classroom was located is also presented in order to show that the broader school learning environment appeared to support the classroom approach to teaching and learning.

According to Marshall and Rossman [1995], some of the fundamental methods relied on by qualitative researchers for gathering information include participation in the setting and observation. The methods of data collection from the classroom selected for this study will now be addressed around these two data collection methods.

Participation in the Setting

Selection of a classroom for this study occurred when the teacher and I discussed the possibility of a study being conducted in her classroom. This discussion took place while we were in a university class together. In these classes, the classroom teacher talked about, what she described as, her inquiry-oriented approach to teaching and learning mathematics and mathematical problem solving. The classroom teacher agreed to the study because she was interested in its focus on problem solving in mathematics. This teacher, hereafter, will be referred to as Lynne, the name she chose in order to address the issue of confidentiality [Gall, Borg & Gall, 1996] for this study. From my discussions with Lynne, it appeared that her classroom setting had the potential to be "information rich" [Gall, Borg & Gall, 1996]. Her classroom would be suitable for the study because [1] entry to the classroom was possible; [2] there was a high probability that a rich mix of processes, people, programs, interactions and structures of interest were present; [3] as the researcher, I was likely to be able to build trusting relations with the participants in the study and [4] data quality and credibility of the study were reasonably assured [Marshall & Rossman, 1995].

In qualitative research, the presence of the researcher in the lives of the participants who are invited to be part of the study is fundamental to the paradigm. Immersion in the setting allowed me to hear, see, and begin to experience reality as the participants do. According to Marshall and Rossman [1995] this raises a range of strategic, ethical and personal issues.

One of these issues is how the researcher plans for the research role. Patton's [1990] continua for thinking about one's role in planning the conduct of qualitative research framed my role for this study. According

to Patton, at one extreme is the full participant who goes about ordinary life in a role or set of roles constructed in the setting. The other extreme is the complete observer who does not engage in social interaction. Marshall and Rossman [1995] state that all possible complementary mixes are available to the researcher but, in most cases, some form of informal participation takes place.

In this study, I primarily used a participant observer role for the collection of the classroom data. According to Spradley [1980] participant observation is an ethnographic technique which is concerned with the meaning of actions and events to the people we seek to understand. Gall, Borg & Gall [1996] describe the participant observer role as one in which the researcher observes and interacts closely enough with individuals to establish a meaningful identity within their group. As a participant observer researcher, I came to the selected classroom with two purposes, as described by Spradley [1980]: [1] to engage in activities appropriate to the situation and [2] to observe the activities, people and physical aspects of the situation.

I used a participant observer approach for this study because such an approach was helpful for: [1] obtaining large amounts of contextual data quickly; [2] facilitating cooperation from research participants; [3] discovering complex interconnections in social relationships; [4] collecting data in a natural setting; [5] obtaining information on non-verbal behavior and communication; [6] collecting data on sub-conscious thoughts and behavior; [7] facilitating validity checks; [8] facilitating discovery of cultural nuances; [9] providing background context for more focus on activities, behaviors and events; [10] uncovering the subjective side of organizational processes [Marshall and Rossman, 1995].

Another issue concerning the presence of the researcher in the lives of the participants, according to Marshall and Rossman [1995], is the extent to which the nature of the study is shared with the participants. This issue was addressed in the following ways: [1] In September, 1994, Lynne told her students that they had been invited to be part of a study about mathematics. She said that she had accepted the invitation for herself and her students because she believed that they could benefit from the study. [2] A letter explaining the study, was sent home requesting parents' and students' consent [see Appendix A].

Lynne discussed with me how this study could be beneficial to her own practice and to the learning of her students. She felt that being part of the study would provide an opportunity for her to discuss her teaching practice and reflect on her teaching in mathematics and mathematical problem solving. She felt that her students could benefit from her own reflection on her practice and from the focus on mathematical problem solving that this study entailed. Coming into Lynne's classroom and being with her students reflected a "reciprocity " model [Patton, 1990] in which it is assumed that "some reason can be found for participants to cooperate in the research and that some kind of mutual exchange can occur" [p. 253]. Jorgensen [1989] also addresses this type of collaborative relationship with participants in an inquiry as one in which they find something that makes the cooperation worthwhile, whether that feeling is one of importance from being observed, useful feedback, pleasure from interacting with the observer, or assistance in some task. In Lynne's classroom, the students initially felt important that they were part of the study and valued the addition of another adult in the classroom. According to Lynne, her students looked forward to the days when the research was being conducted

"just for the extra attention"!

In Lynne's classroom, my role, as the researcher, often became an "additional pair of hands", particularly in the morning when the students arrived at school, as well as at recess and lunch. As suggested by Marshall and Rossman, [1995], interpersonal considerations throughout the research included a conscious effort on the part of the researcher to build trust with the participants by being sensitive to their needs as well as the needs of the research.

Observation

According to Marshall and Rossman [1995], observation entails the systematic noting and recording of events, behaviors and artifacts [objects] in the social setting chosen for the study. There is also the assumption that the observed events, behavior and artifacts are purposeful and expressive of deeper values and beliefs.

While in the classroom, "condensed account " field notes [Spradley, 1980] were taken. Following the classroom observations, an "expanded account" [Spradley, 1980] of these field notes were written. These field notes consisted of the details of what was observed and recalled in the classroom but not recorded on the spot. The field notes also recorded informal conversations with students and the teacher, as well as parents and administrators who visited the classroom.

In the early stages of data collection in the classroom, there was a focus on broad areas of interest but without predetermined categories or checklists. According to Marshall and Rossman [1995], the value of this stage is to discover the recurring patterns of behavior and relationships. During this time I took a more passive role. Spradley [1980] describes this

role as one in which the researcher is present at the scene of observation but does not participate or interact with people to any great extent. In later stages of the data collection, I used a more focused approach in order to check emerging patterns in the data in order to see if they explained the behavior observed over time and in a variety of settings. Qualitative research is defined by this quality of flexibility during the research process. This flexibility allowed patterns in the study to be identified and described through early analysis of the field notes.

On those occasions, when my research role became more active in the classroom by assisting individual, small and large groups of students with their work in mathematics, my observations were later recorded in field notes when the work with the students had been completed.

In addition to the observations recorded, there is a reflective component to the field notes which records the more personal side of the fieldwork. This component of the field notes includes reactions to observations and occurrences and conversations in the classroom as well as experiences, ideas, fears, mistakes, confusion, breakthroughs and problems that arose during fieldwork. Spradley [1980] writes that making such introspective records of fieldwork enables the researcher to take into account personal biases and feelings in order to understand their influences on the research.

In addition to the field notes, classroom data were also collected through a dialogue journal between myself and Lynne, the classroom teacher. This journal used a letter format through which observations and reflections on events in the classroom were exchanged weekly between us. These exchanges took place between December, 1994 and April, 1995 and formed the focus for the informal discussions about the study. The

dialogue journal also provided opportunities for Lynne and I to discuss and reflect on her practice in mathematics which she had indicated earlier would be helpful for her own professional development.

In addition to the field notes and the dialogue journal, classroom artifacts were also collected in the classroom. According to Goetz and LeCompte [1984], artifacts are material manifestations of the beliefs and behaviors that constitute a culture. Artifacts collected from the selected classroom include copies of students' report cards, notices to parents, letters to the teacher from parents, copies of posted notices from the classroom and samples of students' work from their mathematics journals. The artifacts constitute data that indicated people's sensations, experiences and knowledge that connoted opinions, values and feelings. The purpose of the collection of classroom artifacts was to help to develop an understanding of the classroom setting [Marshall & Rossman, 1995].

Description of the School

The purpose of this section of the chapter is to describe the school where this study took place as one that supports Lynne's approach to teaching. The data for this study contained substantial references to the school, the parents and the general classroom. It was felt that such a significant amount of data should be included in the classroom observation The description and documentation that follows consolidates the observations and reflections taken from mid-September, 1994 to April, 1995.

According to the research of Seeley et al [1988], knowledge is not independent but is fundamentally "situated" and a product of the context and culture in which it is developed. The context and culture of the school

will be presented through the description of the physical and organizational design of the school and the classroom, the sense of school and parent community which encouraged independent and responsible learning.

The classroom selected for this research was one of twenty-four classrooms at Hilltop School, the chosen name of the school for this study. Hilltop School is an elementary school for grades 1 to 6 including an Early Childhood Program. The school, with 689 students was built in 1990 to serve a newly forming community of professional families and is one of the largest in the district. Fortunate to have a large volunteer base and a strong financial commitment from parents, it has abundant resources, including an impressive computer lab, a recently completed outdoor indigenous garden and amphitheater, as well as the usual additions that accompany a newly opened school.

At Hilltop School, there is a sense of community and belonging. The school, has been organized into three learning communities, called "family groupings". Each family grouping is housed in one of three sections of the school that radiate out from a central hub where the offices, library, gym, art room etc. are housed. Each "spoke" is named after a local landmark and consists of three inter-connected multi-aged classrooms: Grades 1-2; 3-4; 5-6. The Kindergarten is housed in a separate part of the school. With its skylights, wide hallways and plants, the school atmosphere is open and reflects the community building that has been intentionally created here.

In Lynne's classroom, a large schedule of the week is posted at the front of the room and students are well informed about what will be happening in the classroom. Physical boundaries are clear so that students can see what activities are set in specific spaces. Mailboxes at the door as

well as pocket charts for library cards would suggest that students are encouraged to be responsible for their learning. Windows allow children to comfortably witness the changing drama of nature outside. Desks, arranged in groupings of six, are personal spaces where arrangements of pencils in colorfully decorated containers as well as originally constructed name tags clearly reflect the personalities of the children who work there. Lynne’s desk is moved to one side and faces the wall where it functions as more of a work station rather than as a traditional teacher's desk positioned at the front of the classroom. A movable wall separates this classroom from the adjoining classroom where Lynne’s teaching partner and her students work. Lynne said that she and her partner start the year mostly with their own class grouping of children. As the year progresses and the two teachers come to know their children better, they open the wall between the classes combining the classes more often.

The focus of Lynne’s classroom is the gathering spot in the middle of the room. It is in this place that students meet for whole class briefings, stories and any activities that necessitate the attention of the whole group. From there, students either gather at their table groupings or in small groups seated on the floor.

The outdoor facilities of the school include a very large play area with landscaped contours and massive stone outcroppings as well as the recreational facilities of the adjoining community center which include an outdoor skating rink. The overall appearance of the school, both inside and outside, represents an environment that appears to be attuned to the needs and enjoyment of children and teachers. Hilltop School's statement of beliefs reveals the school's philosophy.

Hilltop School Statement of Beliefs

- Learning occurs at different rates and in a variety of ways.
- Learning is lifelong.
- Learning occurs through life experiences and is best facilitated in a caring and supportive environment.
- Learning involves taking responsibility for goal setting, decision making, choice and ownership.
- Learning occurs best when it is relevant and meaningful.
- Learning can be enhanced in a social environment which involves collaboration, reflection and sharing.
- Learning occurs best in an environment that nurtures self-esteem.

The sense of community being created at Hilltop is seen in school-wide inter-grade gatherings of the "family groupings". An example of such groupings occur when students gathered to work on writing, drama and art within a Halloween theme. The school notice of October 15, 1994 for teachers and students states that the goal of this Halloween activity was to "get to know the members of the family groupings and to appreciate the efforts of everybody".

Parents are an integral part of the learning community at Hilltop and are seen volunteering in many areas of the school throughout the day They were observed relating easily and comfortably to teachers, support staff, administrators and other students. For a mother of a student in Lynne's classroom, this way of being in the school helps her in voicing her thoughts and concerns about her daughter's education. She says that she feels comfortable in contacting the school regarding any information that she needs or concerns she might have. She says that she might not always get the information she wants but feels that she can trust the school staff to do everything they can to help her.

An example of parental and community support at Hilltop School was shown at a Science and Technology Fair held in October of the year in

which this study was conducted. In this school-wide fair, lessons in astronomy, computers, engineering, life sciences, and communications were given mainly by parents of the students. The scope of this endeavor and the personal investment of time and interest on the part of parents was evidenced when a parent of a student in Lynne's classroom brought the family airplane to the school as part of the Science and Technology Fair. The student's father arranged with the local highway patrol to block off the streets and make the necessary arrangements so that the four-seater aircraft could be brought to the school and made available for tours by the children of the school!

In keeping with the sense of community within the school, the classes at Hilltop school are organized in multi-aged groupings [Opuni and Koonce, 1992], an arrangement whereby children of various ages, abilities and interests are placed together in a learning environment. The research of Opuni and Koonce [1992] regard this way of working with children as providing "the opportunities for inter-age challenge, mentoring, support, friendships, leadership, peer and cross-tutoring" [p. 7]. Lynne, the classroom teacher, recently completed a study on multi-aged groupings as part of a Master's degree and part of her reason for choosing to teach at Hilltop was the multi-aging in the school. Lynne says that working with children within this multi-aged framework is a much broader issue than an organizational strategy. She sees multi-aged groupings as a structure that encompasses a wide range of interests, abilities and strengths and helps to place student learning at the centre of teaching. Lynne's understanding of multi-aged groupings reflects the research of Simon [1993] who suggests that recent mathematics education efforts have led to the development of a model of mathematical teaching in which students' thinking and

understanding is taken seriously and given a central place in the design and implementation of instruction.

Lynne says that her beliefs in the pedagogy that underlies the structure of multi-aged groupings enable her to meet the individual needs of students according to their cognitive, emotional, psychological and physical needs. Lynne regards multi-age grouping as enabling children to work at different developmental rates without obvious remediation or retention and without special arrangements for acceleration. In Lynne's opinion, in a multi-aged learning arrangement, children are not taught curriculum according to grades. Rather, ongoing skills and concepts inherent in the curriculum are organized in common themes and adapted to meet the needs of individual students and the requirements of curriculum. Her belief is that children's needs, abilities, interests and styles of learning differ and that the learning environments should reflect this.

Despite some problems when this instructional strategy was first introduced, the parents of students at Hilltop School seem to view multi-aged groupings as a positive learning milieu for their children. Two parents indicated this to me in informal discussions. One parent said that she did not have any problems with the multi-aged groupings because her children had all done well with it. She was concerned, however that, because multi-aged groupings were a relatively new concept, her children would not be prepared for Junior High. Another parent said that this approach appeared to be beneficial to his daughter as it encouraged her keep up with the older students in her class and also, in the next year, she could help the younger students and thus further assist her learning.

Parents and administrators notice that the approach to teaching and learning in Lynne's classroom encourages students to be responsible for

their learning. A parent volunteer commented that she wondered how Lynne knew who was doing what in the classroom! She said that whenever she came into the classroom to help, she noticed that the students were all engaged in what looked like different tasks. She said that Lynne could tell her exactly what each child was doing. She found this amazing and noted that the learning was so much deeper than when she was in school.

A school administrator noted a sense of independence and student responsibility in Lynne's classroom. The administrator commented that she tried so hard to catch those who were working in the hall to see if she could catch them off task when they were out of Lynne's sight. She said that she had never been able to do that and noted that the students were on task whenever she passed by, even when she came from around the corner!

A parent also commented on the atmosphere that encourages student independence in Lynne's classroom when she wrote a note concerning her ill son:

Dear Lynne,
Just a short note to apologize for [son's name] entering the classroom late because of appointments, coughs, etc. It must be disruptive.
I would also like to acknowledge your giftedness in teaching. The children seem very happy and so advanced compared to the fall. [Son's name] just hates to miss school. That has to be a good sign! He appears to be very secure and acting very responsibly. Keep smiling. You are doing a great job!

When Lynne’s name is mentioned among the student teachers who are often in her classroom as part of their field experience, the first comment that arises is that, as a student teacher, it is easy to be with students in her classroom. The student teachers say they notice that

students in Lynne's classroom have a sense of ownership about what happens in the classroom as they take responsibility for daily routines and classroom experiences.

A respect for the feelings and the opinions of her students encompasses Lynne's teaching. She places the needs of her students at the centre of her teaching. She literally does this in the physical arrangement of the desks in her classroom which promotes a sense of community. The students' desks are placed in groups in the centre of the classroom.

In summary. Data Collection #1 which took place over an eight month period in Lynne's classroom is presented below in Figure 2.

Figure 2: Summary Chart of Data Collection #1: The Classroom Data

Technique	Purpose	Details of Collection
Field Notes	To record classroom observations and reflections.	Approximately 200 hours of classroom observation.
Dialogue Journal	To record classroom observations and reflections from the perspective of the teacher and the researcher.	Journals exchanged approximately once a week from December, 1994 to April, 1995.
Artifacts	To help to develop an understanding of the classroom setting.	Artifacts collected included: students' report cards, notices to parents, letters to the teacher from parents, copies of posted notices from the classroom and samples of students' work from their mathematics journal.

Data Collection #2: The Problem Solving Data

Following the first collection of data, four students, selected by Lynne, were audio taped as they worked on three sets of mathematical

problems. The purpose of this second set of data was to explore the problem solving practices of these four students as they worked on routine, traditional problems presented to them [Bottge & Hasselbring, 1993].

The data collection from the problem-solving sessions will address the following topics: [1] Selection and Description of the Students for the Problem Solving Sessions; [2] The Problem Solving Sessions; [3] The Problems Chosen for the Problem Solving Sessions.

Selection and Description of the Students for the Problem Solving Sessions

In September, the classroom teacher, Lynne, and I discussed choosing four students for the problem solving sessions which would take place in May and June of that academic year. The students were chosen for the problem solving sessions by Lynne, based on their growth in mathematics over the current school year and their ability to articulate their thoughts about their learning in mathematics. According to Lynne, the four students chosen for the problem solving sessions had demonstrated growth in mathematics through their deepening understanding of math concepts and their increasing openness in writing and talking about what they were learning since the beginning of the school year. Lynne said that all four students were writing more about what they were learning in their mathematics journals than earlier in the year. In addition, she noted that they also asked more questions in their journals and in their classroom discussions than they had earlier in the year.

The four students chosen for the problem solving sessions were all girls. Two students were in Grade III and two were in Grade IV. The fact

that all four students were girls was not seen as a focus for the study. All four students were in Lynne's class for the first time because this was Lynne's first year in this school.

The students chosen for the problem solving sessions were approached individually by Lynne and I and asked about their participation in the sessions. Each agreed to be a participant. The four students chose the pseudonyms of Christene, Cristel, Lindsey and Karen, for purposes of confidentiality for the study. Although all four students had worked in groups in their classroom, coming together as a group for the problem solving sessions was the first time that Christene, Cristel, Lindsey and Karen had worked together exclusively.

Christene, Cristel, Lindsey and Karen will now be described in relation to their learning in mathematics through observations made during their classroom problem solving sessions as well as through comments made by Lynne and the parents of the students.

<u>Christene</u>, aged eight years and in Grade III, presented herself as a very active participant in the classroom and as a student who openly questioned her peers. Students who worked with her in her classroom mathematical problem solving groups referred to her as the "problem buster", that is, the one who indicated to the group that something had been done wrong. When an answer to a problem had been reached, Christene would often be the one who checked to see, as she stated, "if the answer was reasonable". She said that working on problems in a group was good because everyone took turns reading the problems and if someone read some of the information wrong then the others could help her out. She said that, by working this way, the group didn't work on a

long answer only to find out later that the wrong number was copied.

Lynne commented that Christene had a good understanding of mathematical concepts and was ready to move on to build on those concepts. "She just seems to understand how to go about working with problems. I think she has had a lot of practice with problem solving because she seems very comfortable exploring all the possibilities when she works on her own or in a small group".

Christene's parents discussed the changes they see in Christene's mathematics learning. "We see changes in many areas. The emphasis has shifted to problem solving using your basic facts and functions". Christene's parents expressed support for the way in which Christene was learning "as long as she understands what she is doing and doesn't fall behind".

When asked about being chosen to work in the problem solving sessions for this study, Christene agreed that she wanted to participate because "four heads are better than one".

Cristel, aged eight years and in Grade III, was the youngest of the group chosen for the problem solving sessions and, according to Lynne, a very good mathematics student throughout her previous schooling.

In her classroom mathematics problem solving group, she presented herself as an active questioner who, according to her peers, knew the "right way" to solve problems. In her problem solving groups, Cristel acted as a gatekeeper, keeping all of the members of the group on the right path. Discussing her feelings about working in a group, she said that she liked the way that the group "all put our answers together to see which ones seem reasonable, most appropriate - like when we add up money and ask

each other what we got ".

Cristel said that using calculators helped her in problem solving because she could try different operations and then think about whether the answer was reasonable. She thought that this way of trying out answers is easier than working it all out. She also said that she has not mastered many calculations yet because she is in grade three but feels that she is learning which operations to use because she is working with students in her classroom who are a year older.

Cristel's parents said that they wanted Cristel to "have a good understanding of her basic facts and functions because these are necessary before moving on to other areas such as problem solving". They said that they realized that math has changed greatly since they were in school and they did not expect it to be the same. "Even when we were taking math, it was changing. At that time the 'new Math' was introduced and our parents had problems with that!"

When asked about being chosen for the problem solving sessions, Cristel felt that she would like to work in this group because, in her mathematics class, students worked in groups all the time and they learned how to wait for others and how to talk about what they were doing.

<u>Karen,</u> aged nine years and in Grade IV, was the student whose voice was most obvious in the transcriptions of the problem solving sessions. Her words "hey guys" echoed throughout the data as she addressed the three other students in the problem solving sessions. Taller than the rest and the most social in her daily life, she extended her social nature into her group work!

In her classroom mathematics problem solving groups, Karen

presented herself as one who was eager to engage with others. She said that problem solving was easier in a group because "everyone can put in an idea". In these groups, she appeared to be thoughtful about what she was doing, often hesitating and asking questions. When she confronted an answer or an argument with which she did not agree, she took time to reconsider what she was thinking and what the others were saying. She said that she had found a way to multiply by regrouping the larger numbers into smaller numbers that she could manage. She said that she does this because she does not know the multiplication tables yet so this is how she multiplies large numbers while she is "getting there".

Karen's parents commented that their daughter was a good learner and hadn't had too many problems yet. "As she gets older, I think she will need to keep on top of the math concepts, always understanding one thing before going on to the next. It's easy to get lost if you haven't mastered a particular area".

Despite the fact that Lynne had seen growth in Karen's ability and attitude in mathematics and Karen had wanted to participate in the problem solving sessions, Karen gave the impression that mathematics learning was generally distasteful to her! However, she said that she would like to work with the three other students in the problem solving groups because the group made it fun! "I am very social, you know, and I like to be with my friends, or even people who are not my friends to do math or any subject".

Lindsey, aged nine, and in Grade IV, was the oldest and, physically, the largest student in the group and the most silent. According to her teacher, Lindsey was one of the best mathematics students in her class. Just prior to the taping of the problem solving sessions, Lindsey had received

news that she and her family would be moving to South America within the month. This news had absorbed most of her usual energy and enthusiasm. During the problem sessions, she often sat off to one side playing with her pencil or other things.

In her classroom problem solving groups, Lindsey often presented herself as a "wise sage". In this role, she sat back, giving out advice and often illuminating inconsistencies in the calculations. In the classroom groupings, she often made connections between what was being done in mathematics and the "outside" world. By making statements such as "That's not what you would pay at the supermarket" or "That wouldn't make sense on the playground" she made the connections that she felt were necessary for the mathematical task.

Lindsey's parents supported her learning despite what they saw as significant differences from their own school experiences in relation to mathematics. "Whereas, in my day, memory played a large role in elementary schools, today's students rely far less on memory and far more on understanding the reasons behind mathematical functions. I'm not sure that this approach is the right one. Perhaps, once the memory focuses on a particular mathematical function, the rationale will follow".

Lindsey's thoughts about being chosen for the problem solving sessions were that it should help her to learn because she would be talking to the others in the group about their answers. She also felt that extra practice in problem solving would "come in handy in her new school in Venezuela".

The Problem Solving Sessions

The six problem solving sessions took place once a week for six weeks from May to mid-June, 1995. Each session lasted approximately 45 to 60 minutes. During each problem solving session, the four selected students sat at a round table with the audio tape recorder in the centre. Marshall and Rossman [1995] discuss audio taping as a method of data collection which may be used in conjunction with observation in order to capture an ongoing flow of events, in this case, the students' talk as they worked on the problems. A video tape of the problem solving sessions was also taken but did not form part of the data analyzed due to its limited usefulness.

Lynne agreed that the students could be removed from the classroom to work on the mathematical problems for one morning per week for six weeks during May and June, 1995. She felt that these four students could benefit from the problem solving sessions because the class was working in small groups on mathematical problems and the problem solving learning experience had the potential for more practice in mathematics for these four students.

The six sessions took place in a setting apart from the classroom. The reason for removing the students from the classroom was to provide for a clear audio tape sound in the separate setting away from classroom noise and activity. However, this new setting in a corner of the multi-purpose room, appeared to be almost "experimental". Mercer [1995] states that there is a lot of evidence, especially from the study of children's' cognitive development, that the performance of subjects in experimental tasks is strongly affected by their interpretation of what they are meant to do. In this setting, students were not only working on what is described as

routine, traditional problems but they were now in a setting that could be described as contrived, almost "test-like".

During the audio taping, as the researcher, I played a more passive role as the students worked on the problems. Field notes taken during the problem solving sessions contain a more procedural record of the problem solving sessions and details of the audio taping.

During the problem solving sessions, the students were presented with single sheet copies of the problems. They also had access to paper, pencils and calculators.

The Problems Chosen for the Problem Solving Sessions.

During the six problem solving sessions, the four students were presented with three sets of problems titled: *At the Movies* [contained 7 problems], *Sudden Wealth* [contained 11 problems, but 3 problems, #9, #10 #11 were not attempted by the students] and *Carpet Sale* [contained 2 problems]. In total, twenty problems were presented to the students, of which, they attempted seventeen problems.

The three sets of problems presented to the students were routine and traditional [Bottge & Hasselbring 1993]. Bottge & Hasselbring [1993] describe such problems as routine because they appear to connect with real life experiences but students do not see such connections. The first set of problems, *At the Movies*, is about a family going to the movies. The second set of problems, *Sudden Wealth*, is about winning money. The third set, *Carpet Sale* is about buying carpet for a house. Such routine problems describe situations textually, on the surface, rather than contextually, that is arising out of real situations. Such problems are also geared to specific operations and concrete answers [Bottge & Hasselbring,

1993]. All problems presented to the students exemplify this request for specific answers by asking "how much?" "how many?" "what is?" "what will?" or "estimate"

Routine problems are described by the *Standards* [1989] as problems that are ready-made exercises with easily processed procedures and numbers which provide context for particular formulas or algorithms. Such problems usually require only one step to solve [Becker, 1990].

The problems chosen for the problem solving session are similar to the types of problems that teachers often keep in their classroom files and present to students and are also similar to the types of problems often found in mathematical textbooks.

The problems for the problem sessions were chosen from the researcher's files. Lynne reviewed the problems to make certain that they were not too difficult for the students.

Part of the reason why the problems were chosen was because they were similar to the types of problems that teachers might choose. Such problems appear to address the interests of the students.and, therefore, appear to be aligned with the NCTM "vision" for problem solving. By choosing such problems, teachers believe that they are teaching problem solving in an inquiry-oriented approach.

The three sets of problems are presented below. The complete pages as presented to the students are contained in Appendix B.

At the Movies

1. Tom had $10 when he went to the movies. His ticket cost $3 75. He bought popcorn for $1.50 and a soft drink for $0.95. How much money did he have left?

2. M. Hamelin bought 3 boxes of popcorn at $0.95 each and a package of peanuts for $1.20. How much did he spend?

3. The 2 adults and the 2 children of the Neilson family went to the movies. How much money did the tickets cost?
[Adults: $6.50; children: $3.75]

4. One evening the popcorn concession sold 275 small boxes, 183 medium boxes and 56 large boxes of popcorn. How much money did it take in?
[Large popcorn: $2.25; medium popcorn: $1.50; small popcorn: $2.25]

5. The popcorn machine uses 12 L of cooking oil a week. The oil costs $3 a litre. How much does the oil for 52 weeks cost?

6. One evening the popcorn concession took in $228 for small boxes and $279 for medium boxes of popcorn. How many boxes of popcorn did it sell?
[Large popcorn: $2.25; medium popcorn: $1.50; small popcorn: $2.25]

7. One evening the food concession sold 126 regular-size soft drinks and 77 large size. How much more money did it take in from the large-size drinks.
[Large soft drink: $1.50; regular soft drink: $0.90]

Sudden Wealth

Problem 1
A mysterious genie has just told you that you will receive $3 a minute for the next year.
How much will you receive?

1. in one hour ____ 2. in one day ____

3. in one week ____

4. in one year ____
[52 weeks]

Problem 2
With your new wealth you decide to buy a cassette tape for each student in your school. Suppose there are 746 students in the school. Each cassette costs $8.95, plus $0.64 tax.

5. How much do the cassettes cost without the tax? ____
6. How much tax will you have to pay? ____
7. What will the total cost be? ____

Problem 3
You decide to treat everyone in your class to lunch at Burger Palace. There are 29 students in your class. Big Burgers cost $1.79, soft drinks cost $0.75 and sundaes cost $1.35. Each student gets one of each.
What is the cost?

8. all the burgers? ____
9. all the soft drinks? ____
10. all the sundaes? ____
11. the total? ____

Carpet Sale

The floor plan shows the main floor of a house
Each small square represents one square metre.

1. Estimate the cost of the carpeting using each kind of carpet. [Velvet: $19 a square metre; Royal: $28 a square metre; Plush: $23 a square metre; Harmony: $13 a square metre].

   Harmony carpet ____ Velvet carpet ____
   Plush carpet ____ Royal carpet ____

2. How much would it cost to use Velvet carpet in each room? Show how you got your estimate.

Kitchen	Family Room	Dining Room	Living Room

A summary of Data Collection #2 is presented in Figure 3

Figure 3: Summary Chart of Data Collection #2: The Classroom Data

Technique	Purpose	Details of Data Collection
Audio taping	To record problem solving sessions	Audio taping of six problem solving sessions of 45-60 minutes each.
Field notes	To record observations and reflections during the problem solving sessions.	Six observation sessions

Overview of Data Analyses

Marshall and Rossman [1995] refer to qualitative data analysis as the search for relationships among categories of data and the process of bringing order, structure and meaning to the mass of collected data.

The purpose of analyzing data from the selected classroom and the problem solving sessions was to explore and document the nature of the teaching and learning that occurred in the classroom and the problem solving practices of the students chosen from that classroom. The field notes, dialogue journal, artifacts and transcriptions of the audio tapes provided the raw data for the analysis.

According to Marshall and Rossman [1995] the most fundamental operation in the analyses of the qualitative data is discovering the significant classes of things, persons and events and the properties that characterize them.

One of the characteristics of the qualitative research methodology used for this study was the flexibility to analyze the data during the process of the data collections as well as upon completion of the data collections. In qualitative studies, data collection and analysis can occur simultaneously in order to promote the emergence of theory grounded in empirical data [Glaser and Strauss, 1976; Vidich, 1969]. The early analyses of both sets of data allowed for initial hypotheses which suggested categories and patterns around which subsequent data could be gathered. As suggested by Marshall and Rossman [1995], the two data analyses were complete when categories, and patterns were defined and the relationships between the categories and the patterns established.

Analysis #1: The Classroom Data

The process of analysis entailed reading and reviewing the classroom data in order to become familiar with it. During the reading process, as suggested by Marshall and Rossman [1995], the data available were listed, minor editing was performed to make the field notes retrievable and there was a general "clean up" [Pearsol, 1985] of what seemed rather overwhelming and unmanageable.

Analysis #2: The Problem Solving Data

The process of analysis of the problem solving data entailed, once again, reviewing in order to become familiar with the data. From the

review of the data, events which are described by Marshall and Rossman [1995] as the noting of regularities in the data, were elicited. The events were categorized to describe the problem solving practices from the data. These categories were defined, labeled and colour-coded to distinguish them from each other and framed by specific guidelines to determine whether each event was or was not an instance of a practice.

As the categories of meaning emerged, attention was paid to internal convergence and external divergence, that is, that the categories should be internally consistent but distinct from each other, as suggested by Guba [1978].

A summary of the data analyses is presented below in Figure 4.

Figure 4: Summary Chart of the Data Analyses

Step 1	Step 2	Step 3
• Initial analysis of the data • Development of preliminary hypotheses leading to possible themes around which the data could be gathered. • Familiarization with the data	• Multiple readings and study of the data • Identification of events which share sufficient similarities. • Generation of a category for the classroom data and five practices for the problem solving data • Check for internal convergence and external divergence	• Testing emergent hypothesis • Searching for alternative explanations for the two categories of classroom data and five practices of the problem solving data

Trustworthiness

All research must respond to canons that stand as criteria against which the trustworthiness of the project can be evaluated. Lincoln and

Guba [1985] refer to this as establishing the "truth value" and have proposed constructs that reflect the assumptions of the qualitative paradigm. The trustworthiness of this study is framed within Lincoln and Guba's constructs of credibility and transferability.

Credibility

According to Lincoln and Guba, this first construct of credibility has as its goal to demonstrate that the inquiry was conducted in such a manner as to ensure that the subject was accurately identified and described. Merriam [1988] suggests that credibility, also referred to as internal validity, is concerned with whether the findings capture what is really there. Suggestions for credibility include triangulation, feedback from the participants and level of involvement with the participants as important to establishing internal validity within the qualitative paradigm.

Triangulation. This study used triangulation or bringing more than one source of data to bear on a single point [Marshall and Rossman, 1995] to strengthen its trustworthiness. Including the classroom teacher provided another angle from which to gain insight into the beliefs and practices of the teaching and learning of mathematics. Using observations, which included informal interviews, reflective notes, a dialogue journal and audio tapes in the two data collections provided different avenues for exploring the beliefs and practices of the teacher and the students in the study.

Feedback from the participants Merriam [1988] describes this suggestion for credibility as taking the data and interpretations back to the people from whom they were derived and asking them if the results are plausible.

In this study, weekly meetings with Lynne, the classroom teacher, during and upon completion of the two data collections provided the opportunity to review together the data and the interpretation of the data so that the observations, including the transcriptions, could reflect what was really happening in the classroom and during the problem solving sessions.

Level of Involvement With the Participants. Merriam [1988] suggests long term observation at the research site. This study took place over a ten month period with approximately 200 hours of classroom observation and approximately 6 hours of audio taping . The amount of direct contact with the teacher and the students allowed for authenticity of the classroom interactions which has the potential to increase the validity of the findings.

Transferability

The second construct for the trustworthiness of this study is that of transferability. Merriam [1988] refers to this construct as external validity which refers to the generalization of the findings from a study. In reference to this study, this means the concern with how the findings of this study can be applied to other situations [Cresswell, 1994]. The procedures chosen for this study enhance the potential for the generalization of the findings [Guba, 1981]. These procedures include Merriam's [1988] suggestions which entail [1] providing a rich, thick description, in this case of the school, the classroom, the teacher and the students to provide a base of information for anyone interested in transferability and [2] establishing that the theories for data collection are guided by concepts and models, in this case by the suggestions of multiple researchers and writers.

Ethical Considerations

This study had no potential for producing physical or mental harm to the participants. All participants were informed about the purpose of the research and the use of the data obtained from it. The researcher obtained the written consent of each student and his/her parents before starting the study [see Appendix A].

The anonymity of the participants is protected at all times by the use of pseudonyms. The participants were informed of this provision for anonymity, confidentiality and their right to withdraw without penalty at any stage of the study. In addition, ethical clearance was obtained from the university and the school district in which the school was located.

Summary

In this chapter, the design of the study was presented within the framework of a qualitative field study. The development of the study was traced from the initial entry into the classroom in mid-September, 1994, to the audio taping sessions which took place in May and June, 1995. Procedures for the two data collections were discussed. These procedures included observations and reflective notes collected over the ten months of the study and the audio taping of the six problem solving sessions. Procedures for analyses of the two sets of data included analytic, inductive analysis during and upon completion of the data collections.

This chapter also contained discussions around the trustworthiness of the study. Credibility for the study was established through the processes of triangulation, feedback from the participants and the level of involvement with the participants in the study. Transferability of the study was established through data that provided a rich, thick description and

theories for data collection that were guided by concepts and models of multiple researchers and writers.

An analysis of both sets of data will be presented and discussed in the following two chapters.

CHAPTER IV

THE CLASSROOM DATA
FINDINGS AND DISCUSSION

This chapter addresses the analysis of the classroom data. The purpose of this set of data was to explore and document the nature of mathematical inquiry in Lynne's classroom.

As discussed in Chapter III, this first set of data for the study were collected in a selected Grade III/IV classroom over an eight month period from mid-September, 1994 to April, 1995. The written observations and reflections from the classroom were recorded in field notes and a dialogue journal. Classroom artifacts were also collected. The data were then analyzed and grouped into topics in order to document the nature of mathematical inquiry in the selected classroom.

In the very early stages of this study, as discussed in Chapter III, the classroom teacher, Lynne, described her approach to teaching and learning of mathematics and mathematical problem solving as an inquiry-oriented approach. Her description of this inquiry-oriented approach included her encouragement of her students to: [1] use their previously held understandings in order to make sense of their learning; [2] explore and construct mathematical understandings and [3] discuss their mathematical learning with their peers.

Lynne's description of her inquiry-oriented approach reflects the research of Tougaw [1993] who describes such an approach to teaching mathematics as an "open" approach. Tougaw's "open" approach encourages students to: [1] use previously learned knowledge, skills,

beliefs, attitudes and thought processes; [2] search for interesting methods of reaching problem solutions and [3] participate in discussions of the mathematical quality of different solutions to mathematical problems.

The NCTM *Standards* [1989] also reflects Lynne's description of her inquiry approach. The *Standards* [1989] state that teaching and learning in mathematics and mathematical problem solving should reflect the understanding that: [1] children come to mathematics problem solving with "considerable mathematical experience" [*Standards*, p. 16] and that students approach each new mathematical task with previously learned knowledge, attitudes and skills; [2] "children are active individuals who construct, modify and integrate ideas by interacting with the physical world, materials and other children" [p. 17] and [3] children should " share their thinking and approaches with other students and with teachers" [p. 23].

Lynne's description of her inquiry-oriented approach, Tougaw's [1993] "open approach" and the NCTM *Standards* [1989], together, suggested three topics for the presentation of the findings of the research in relation to the nature of mathematical inquiry in Lynne's classroom: These topics, which will be discussed in the following sections of this chapter are: [1] Drawing On Students' Previously Learned Mathematical Attitudes, Skills And Thought Processes; [2] Mathematical Exploration And Discovery, and [3] Mathematical Discussion.

Included in the presentation and discussion of the findings under each of the three topics named above, are those mathematical teaching practices and strategies that, for purposes of this study, are referred to as the characteristics of mathematical inquiry in Lynne's classroom. At the end of each of the three sections, a chart is presented which summarizes the characteristics of mathematical inquiry in Lynne's classroom.

Drawing On Students' Previously Learned Mathematical Attitudes, Skills and Thought Processes

The first way in which Lynne describes her inquiry-oriented approach to teaching and learning of mathematics and mathematical problem solving is as an approach that draws on students' previously learned mathematical attitudes, skills and thought processes. In discussion, Lynne states that her topics in mathematics are arranged to build on the mathematical intuitions that students bring with them to her mathematics class. She also says that her teaching methods are adapted to the notion that her students are interpreters and constructors of mathematical learning theories as opposed to being absorbers of ready made rules and formulae. Lynne's approach is supported by the research of Garofalo [1993], which suggests that successful problem solvers approach problems by trying to construct meaningful relationships and meaningful plans of action.

Lynne says that one way she builds on what students already know is by trying to arrange her curriculum so that it connects with the students' school year and with topics that are familiar to her students. Lynne's approach of building on topics that are familiar to students is supported by the NCTM *Standards* [1989] and *Professional Standards* [1991] which state that teachers should have knowledge of students' interests and experiences. A typical example of Lynne's building on what students already know is presented below:

<u>Lesson A: Who Walks to School?</u> Lynne usually starts the school year with what is very familiar to the students. One of Lynne's first problem solving lessons in data management is to tally and graph which students

walk or bike to school and which students come in a vehicle. Her lesson follows the following pattern:

[1] She starts the lesson by talking about how she drives to school in the morning because she lives a distance from the school.

[2 She asks the students to list the different ways that they come to school. She combines the students' lists on the board.

[3] She asks the class how they could find out exactly how students come to school. Suggestions are listed on the board.

[4] She models a tally and a bar graph on the board.

[5] The students, in their problem solving groups, create tallies and graphs of students' means of transportation to school. Lynne circulates through the groups and makes anecdotal records for her evaluations.

[6] Each group presents their graph to the class for discussion.

[7] The whole class generates a synthesis statement from the data which is written on the board.

[8] Students write in their mathematics journals.

[9] Each group is now invited to make a graph of some other event. [Topics chosen include students' choice of ice-cream flavours, computer games etc.]

Following data management lessons, such as the example above, Lynne teaches basic facts families and mathematical patterning. She does this through mini-lessons on numeration using *Base Ten Blocks* and "lots of games at the beginning of the year". Lynne says that her students "build numbers with blocks all the time. If they don't get basic numeration, they won't get the big number systems". An example of one of Lynne's mini-

lesson on *Base Ten Blocks* follows.

Lesson B: Mini-Lesson On Addition Using *Base Ten Blocks:*

[1] Lynne starts the lesson by placing *Base Ten Blocks* in the centre of each problem solving group.

[2] She allows time for the students to explore the blocks.

[3] She then models addition of 3+4 with the *Base Ten Blocks* on the overhead projector. She does this by placing 3 blocks on the overhead and then 4 blocks, asking "How many do we have here? and here? now all together?"

[4] The students are invited to add, for example, 4+2, using the blocks.

[5] Each problem solving group is asked to explain what they did and learned.

[6] Each group is given a sheet where they record a series of additions done with the blocks. During this time, Lynne is circulating through the groups to ask questions and make observations.

[7] Upon completion of the lesson, the students synthesize the lesson by writing in their journals.

[8] Lynne presents each student with a booklet of addition algorithms. She allows them to use blocks, if they wish, but most students work on the booklets without using the blocks.

Lynne says that she uses these types of lessons early in the year because they provide the opportunity for her to see what the students already know about numeration. She also feels that this experience with blocks is easy mathematics material and encourages students to take risks. The *Professional Standards* [1991] states that one of the roles of teachers in

relation to risk taking in mathematical problem solving is to let students struggle with a difficulty. Additionally, years of research on the role of teachers in problem solving [Polya, 1981; Silver, 1982; Good, Grows & Ebmeier, 1983; Lester, 1985; Kilpatrick, 1985; Becker & Shimada, 1993], suggests that teachers should provide a safe, congenial environment for students to practice problem solving. This research appears to support Lynne's encouragement of her students' risk taking in mathematical problem solving.

Following these mini-lessons on addition and subtraction, Lynne introduces the class to multiplication and division. Throughout the school year, the mini-lessons on basic facts reflect a similar pattern as they focus on other mathematical operations. It was observed that, although Lynne states that problem solving is important for students' mathematical learning, she is challenged in teaching when routinized rules should come first and when they should not. Lynne comments that she will give the students the rules, if those rules will help students see the "bigger picture" better. She comments: "I have given them the rules, now they can move along without being bogged down in the numbers". Goos [1996] indicates that one of the aspects of a teacher's role that is important in establishing a community of mathematical practice is introducing tools for mathematical communication. Goos's [1996] research would suggest support for Lynne's practice of teaching "basic facts" and giving her students the "tools to problem solve".

It was observed that, when Lynne introduces a new topic, she starts by encouraging her students to talk about what they might already know about the topic. An example of this concept is shown in Lynne's introduction of the geometry unit presented below.

Lesson C: Geometrical Shapes:

[1] Lynne introduces the lesson: "We're going to talk geometric terms here. Math has a language all its own and you need to learn this so you can have the tools to describe all the wonderful things you are going to learn."

[2] After saying this to her students, Lynne moves back from the centre of the class circle. In the discussion that follows, one student says that geometry is the "study of shape". Another student says: "The world is made of shapes. The world is a sphere. The classroom is a square." Lynne makes note of these observations on a whiteboard.

[3] The students now go to their problem-solving groups and make lists of all the shapes they can think of.

[4] Lynne adds these to the class list.

[5] Lynne asks the students to create a personal list of shapes and add to these lists for homework. She tells her students that, if they know the geometric terms for these shapes, they should use them. If they do not, they should draw the shapes and share with others in their groups who might know them. She indicates that she will talk about the lists that the students bring to the class next day.

As part of acknowledging students' previously learned mathematical attitudes, skills and thought processes, it was observed that Lynne's approach also encourages students to respond to mathematics in a personal and thoughtful way. An example of this approach is presented below.

Lesson D: Going to Camp: The following problem was presented to the

students as part of a discussion about the students, teachers and chaperones from Hilltop School going on a field trip in the coming months.

> If the buses we have booked for outdoor camp each holds 100 people, how many buses will we need to take 323 people?

In a small-group discussion, a student from Lynne's classroom talks with her peers about this problem:

> You need 4 buses because each bus holds 100 people. The first number is three so you would need 3 buses for 300 people so to hold the rest of the 23 people you would need one more bus. I would like to be on the last bus!

The final statement "I would like to be on the last bus" is an interesting and telling one. The data suggests that this student's comment indicates that it is quite all right to have personal insight into a mathematical problem. She does not simply state that the remaining 23 people [323 - 23 = 3 23/100] would be on the fourth bus but thinks of herself in that situation and possibly thinks of how much more room she would have in comparison to the people traveling on the full buses of 100 people! As this problem solving incident happened in April, it is possible that this student felt that the "permission" to experience real thoughts and feelings in relation to mathematics had come from her mathematical experiences with Lynne over the past eight months.

In this section, discussion has focused on the research findings in relation to students drawing on previously learned mathematical attitudes, skills and thought processes. This aspect of Lynne's inquiry approach, as

presented in Lessons A, B, C and D, is supported by the research of Tougaw [1993], as discussed earlier, and the NCTM *Standards* [1989]. In addition, the research of Novak [1977] would appear to support such an approach by stating that problem solving involves the reorganization of stored information and accumulation of new ideas into the cognitive structure of the problem solver to reach a specified goal. The research of Owens [1989] also suggests that previously held mathematical knowledge has a direct, significant effect on students' mathematical problem solving.

In Lessons A, B, and D Lynne presents topics for mathematical problem solving that are, for the most part, familiar and relevant to her students. The topics presented, such as coming to school, going to camp and creating personal lists of geometric shapes appear to connect school mathematics with students' "real world" experiences of mathematics. Lynne also encourages her students to be active participants in their learning by creating an environment that encourages group work in which students can voice their ideas and thoughts with their peers as well as with herself. As Lynne's students talk among themselves, she circulates from group to group observing her students and listening to what they are saying. She encourages her students to explain their answers, to synthesize and extend their learning. Presenting topics that are familiar and relevant to students and providing opportunities for students to share their thoughts about mathematics are characteristics that would support mathematical inquiry [Nagasaki & Becker, 1993; Becker, 1992; Becker, 1993; Becker & Shimada, 1993].

It was also observed that Lynne models strategies for her students. In Lesson A, *Who Walks to School?*, she models tallies and bar graphs. In Lesson B: *Base Ten Blocks,* Lynne models addition with *Base Ten Blocks*

on the overhead projector before inviting students to add, using the blocks. As mentioned earlier in this chapter, the research of Goos [1996], suggests such characteristics of Lynne's practice as her modeling of mathematical thinking are particularly important in establishing a culture of mathematical sense making.

Although the research of Goos [1996] suggests support for Lynne's modeling strategy, there is an another interesting perspective to this characteristic. This perspective is the possibility that Lynne's students may interpret her modeling of tallies, graphs and *Base Ten Blocks* in ways that could be potentially problematic for mathematical inquiry. The research of Frank [1988] and Garofalo [1989] suggests that one of students' beliefs about mathematics is that the teacher and the text-book are the only sources of mathematical truth and correctness. Lynne's modeling of tallies, graphs and *Base Ten Blocks*, although seen as supportive of mathematical sense-making [Goos, 1996] could also have the potential to be problematic because Lynne's students could interpret her modeling strategy as being the one and only way to organize data about how their peers come to school or as the only way to arrange the *Base Ten Blocks.* Kilpatrick [1985] would say that this type of problem solving is an imitative model where students are trained to model a master problem solver.

In addition, Becker [1990, 1993] and Stonecipher [1986] present another perspective in stating that many mathematical problems that appear in classroom activities are not mathematical problems at all but are routine exercises with a new name. If we consider the four Lessons A, B, C and D presented to students, Lynne had predetermined an "objective" for each of these lessons i.e. to teach tallies and graphs [Lesson A]; numeration [Lesson B], geometric shapes [Lesson C], and number operations [Lesson D].

According to Lester [1980] and Kantowski [1977], the lessons described above are not problems but are practice exercises. Lester [1980] defines a problem as a situation in which an individual or a group is called upon to perform a task for which there is no obvious algorithm which could be used in finding a solution. Kantowski [1977] defines a problem as a situation where an individual encounters a question he cannot answer in a situation he is unable to resolve using the knowledge immediately available to him. Although Lynne does not refer to these lessons specifically as problem solving, she does describe them as part of her inquiry approach. Representing these exercises as problems could be problematic for mathematical inquiry.

In Lesson B, *Base Ten Blocks,* Lynne says that she uses this type of lesson early because such lessons provide an opportunity for her to see what students already know about numeration. She also says that, if students do not get numeration, they won't get the big number systems. It was also observed that Lynne starts the year with addition and subtraction and moves on to multiplication and division, often giving mini-lessons and mini-lesson reviews of basic numeration. It was observed that rules for numeration and geometric terms were presented to the students in order to, in Lynne's words, "help students to see the bigger picture later". Lynne says that she struggles with when she should teach routinized rules and when she should not. Garofalo [1989] states that the nature of the classroom environment in which mathematics is taught, strongly influences how students view the subject of mathematics, the way mathematics should be done and what students perceive as the appropriate responses to mathematics problems. The characteristics of Lynne's "seeing what students already know", her linear approach to teaching addition and subtraction,

multiplication and division, often giving mini-lessons and mini-lesson reviews of basic numeration, and her presentation of rules, as presented above, could reinforce students' beliefs that mathematics computation and doing mathematics means memorization and following rules and the goal of doing mathematics is to obtain the "right" answer [Frank, 1988]. Additionally, these characteristics might be problematic or even non-supportive for mathematical inquiry.

Another area that could be problematic for mathematical inquiry occurs in Lesson D: *Going To Camp*. The problem presented to the students asks "How many buses will we need? Such problems, often introduced by phrases such as "How many?" or "How much?" are referred to by Becker [1990, 1993]; Stonecipher [1986]; Polya [1963]; Bottge & Hasselbring [1993] as routine, number-oriented or traditional problems. These number-oriented problems, according to Bottge & Hasselbring [1993] appear to connect with real life experiences but, in reality, seem artificially geared to specific operations and single, correct answers. Frank [1988] states that one of students' beliefs about mathematics is that mathematics is computation and that mathematical problems are easily solvable in just a few steps. By presenting such number-oriented problems to her students, there is the possibility that this characteristic of Lynne's mathematical inquiry also has the potential to be problematic for mathematical inquiry. Cobb [1984] states that students beliefs are developed from the perceived expectations of the teacher and that these beliefs often do not reflect healthy and realistic views of mathematics. Lynne's presentation of problems that appear to focus on numbers, despite her intentions of encouraging an inquiry approach, could again reinforce students' beliefs that the goal of doing mathematics is to obtain the "right"

answer [Frank, 1988]. This characteristic will be discussed further in the next section when a second example of Lynne's presenting a number-oriented problem to her students is discussed.

In this first topic, drawing on students' previously held mathematical attitudes, skills and thought processes, characteristics of mathematical inquiry in Lynne's classroom have been presented. Some of the characteristics of mathematical inquiry presented are supportive of mathematical inquiry. Some of the characteristics presented have the potential to be problematic for or even non-supportive of mathematical inquiry. These characteristics are summarized in Figure 5 and will be addressed further at the end of this chapter.

Figure 5: **Summary of Characteristics of Mathematical Inquiry in Lynne's Classroom**

Topic 1: Drawing on Students' Previously Learned Mathematical Attitudes, Skills and Thought Processes

Lessons	Characteristics Supportive Of Inquiry	Characteristics Potentially Problematic For Inquiry	Characteristics Non-Supportive Of Inquiry
Lesson A Who Walks to School?	Topic is familiar for students. Students ideas and comments are encouraged . Students are invited to extend the learning activity. Lynne circulates, asking questions and making observations.	Lynne models tallying and graphs.	
Lesson B Mini-Lesson On Addition Using *Base Ten Blocks*	Lynne allows time for exploration. Students are encouraged to explain their answers. Students synthesize their learning in their journals.	Lynne models addition with *Base Ten Blocks.* Lynne sees this type of lesson as a chance to see what her students already know about numeration.	Lynne presents each student with a booklet of algorithms. Lynne introduces the for operations in a linear way. Lynne teaches students routinized rules.

Figure 5 [continued]: **Summary of Characteristics of Mathematical Inquiry in Lynne's Classroom**

Topic 1: Drawing on Students' Previously Learned Mathematical Attitudes, Skills and Thought Processes

Lesson C Geometrical Shapes	Lynne moves from the centre of the circle to allow students to speak. Students compile their own lists of geometric shapes. Students are encouraged to assist each other with the names of shapes.	.	
Lesson D Going to Camp	Topic connects with students' real experiences. Student talks from a personal stance.	Problem presented to students is number-oriented: "How *many* buses will we need?"	

Mathematical Exploration and Discovery

The second way in which Lynne describes her inquiry-oriented approach to teaching and learning mathematics and mathematical problem solving is as an approach that encourages students' mathematical exploration and discovery. Lynne commented: "Problem solving, it's all we do!...all day long!" Such an emphasis on problem solving is supported by the research of Greenes & Schulman [1982]; Suydam [1982]; Thompson & Ratmell [1988] who suggest that teachers are being encouraged to make problem solving a priority.

In the problem solving groups that Lynne has organized in her class, she encourages her students to be independent as they explore mathematical concepts. It was observed that students appear to know what is going on and why they are doing what they are doing.

One example of a problem solving lesson that demonstrates a focus on exploration occurred when the students and Lynne were engaged in a story entitled *Moira's Birthday*. Lynne presented this story to the students in order to explore how the mathematical concept of addition and subtraction were used in the "storied" setting of a child's birthday party. While reading *Moira's Birthday*, Lynne encouraged her students to count with her the number of birthday guests as they arrived at the party.

Lynne and the students together explored the mathematics concepts. What was observed during the reading of the story was Lynne's open and honest attitude to the activity. There was no sense that Lynne positioned herself in a position of authority and expected that the students would try to figure out what she was thinking. Rather, there was a shared sense of discovery for all involved as the students and Lynne talked about how the children in the story used counting to make the story happen. The problem solving lesson is presented below:

Lesson E: *Moira's Birthday*:

[1] Lynne reads the story to the students. As she reads, she uses such expressions as "I wonder what will happen next?" and "That makes six children who have arrived at the party. I wonder how many more will come?"

[2] Following the reading and discussion of *Moira's Birthday*, Lynne encourages her students to create plans for their own birthday

parties. She says "It sounds like Moira had a great birthday. Now, how would you like to prepare for a birthday party of your own? What kinds of things would you need to think of?"

[3] In their problem solving groups, the students are encouraged to [a] make a guest list; [b] plan the birthday menu, including the kind and amount of food, price per item and total cost of food; and [c] plan the party activities, including party games and prizes.

[4] Each group then presents their birthday plan to the class, using large sheets of paper, overhead transparencies and any other materials that could help them to share their ideas with their classmates. Lynne comments that there was so much Math going on that it made her head spin! As the students share their ideas, Lynne makes anecdotal notes about the students learning and the problem solving lesson in general.

[5] Lynne ends the lesson with general comments about how hard the students had worked and how much fun it had been to hear their ideas for birthday parties. Lynne mentioned a few days later that one of the students' parents had actually used the birthday party plan for her own daughter's birthday!

Another example of a problem solving lesson that encourages the students to explore mathematical concepts of numeration, addition and subtraction took place when Lynne distributed calendars of the current year to each problem solving group.

<u>Lesson F: Birthday Calendars</u>:

[1] Lynne invites the students to mark the birthdays of the other students

in the class on the calendars. Lynne's face lights up as the students busily and excitedly moved around, gathering bits of information from each other as they recognized and read aloud the numbers on the calendars.

[2] After the information is gathered, each group is asked to find out how many days until their own birthdays. It is observed that one student tries to count the number of days from the current date to her birthday one day at a time. Then she tries to count "by two's". She says that the reason for this change from counting by one's to counting by two's was to "make the counting faster". Lynne suggests to her to try counting by seven days at a time. "Oh yes, a week" comments one student.

[3] In this problem solving session, most of the students find the number of days between the current date and their birthday by counting each day, or by the week. The discussion is rich and insightful. One student counts by the month [30 days!] because it is "easier and faster" according to him. However, this student also says that he "remembers that some months have 31 days!"

[4] Lynne concludes this lesson by asking the students to turn to a partner and talk about what they had learned that day. She said that it is important that she chooses these "real life" mathematics activities that excite the teacher as well as the students!

An incident with the mathematics "tubs" is very telling of Lynne's exploratory approach to mathematics and mathematical problem solving.

<u>Lesson G: Geometry "Tubs"</u>: A group of teachers and parents from the

previous year had developed "tubs" of mathematics for several classrooms at Hilltop School. The set of "tubs" consisted of plastic bins, each containing a variety of activities designed to build geometrical concepts through a "hands on approach". Lynne had arranged to use the geometry tubs in her classroom but, when they arrived, she was upset to see that each tub was graded Challenging, More Challenging and Most Challenging. Lynne said that she did not think that the labeling was suitable for her class. “I took the labels out. That’s limiting kids. The kids know what they can do. When you limit what students can learn , they will not try”.

In contrast, Lynne developed another problem solving lesson by organizing a set of tubs for an integrated mathematics, science and technology unit in her classroom. In these tubs were assorted gears and pulleys, wood and building materials and building pieces. Her instructions to the students about the tubs were to “build something that moves”. Lynne commented on this: "I think that this group are all such keen learners. They want to try it all. Learning is very exciting for them. I love watching kids' excitement over learning - my first goal of teaching." She says that she notices that students working in this way that encourages exploration and discovery know what has to be achieved. It was observed that the students spent their time productively, using resources appropriately and asking for assistance only when the resources proved inadequate.

The problem solving lessons presented above are indicative of what happened in Lynne's classroom on a daily basis. Often the lessons were integrated with other subjects and extended over several days, a characteristic supported by the NCTM [1989] which suggests that students need to work on problems that may take hours, days and even weeks to

solve.

In some cases, the lessons took unusual forms as Lynne tried to incorporate problem solving into other areas of the curriculum as in the following example:

Lesson H: Photographs: On one occasion, Lynne took photographs of the students while they worked on their mathematics. She then placed the photos that she had taken on pieces of poster board and invited the students to write about what was happening in the pictures. She commented that she was very impressed with the quality of the writing in this activity and about how much the students knew about their mathematics program. One student wrote in her mathematics journal about what she was doing in her photograph. Inherent in what she wrote was her sense of exploration and discovery as she grouped *Base Ten Blocks.*

> I am grouping the longs [units of 10] and tinys [units of one]. This is because I am trying to figure out how to build multiplication. I know that I started with a small group of 15 and then I had to see what it would look like if this 15 was expanded 3 times. I got it wrong the first time. I put a pile of 15, 1 long [units of ten] and 5 tinys [units of one], and then I put 3 tinys [units of one] beside that. It just didn't look or feel that it was right. Then I made up a story like there was a gardener who had to put 3 gardens in a park. He wanted 15 trees in each garden and needed to find out how many trees to order from the tree ordering store. I got it right this time but I needed a story to get there.

Lynne said she was convinced that the reason why the students wrote such good descriptions of what they were doing in the photographs was that the students see the mathematics program as their own. Another student in Lynne's class seemed to confirm this when he wrote in his journal that the

students were really proud of their work with these photographs because they all had a part in the writing. Lynne commented that this experience had been exciting for her too because she loves to see students learn and build meaning.

In this section, discussion has focused on the research findings in relation to students' exploration and discovery in Lynne's inquiry-oriented approach, as presented in Lessons D, E, F, and G. This approach is supported by the research of Tougaw [1993], and the NCTM *Standards* [1989], as discussed earlier in this chapter. In addition, such an approach is supported by the research of Becker [1992, 1993], Becker & Shimada [1993] who state that mathematical problems should encourage mathematical exploration, critical thinking skills, higher-order thinking, discovery of mathematical concepts and problem solving strategies.

In Lesson E: *Moira's Birthday*, and Lesson F: *Birthday Calendars* Lynne presents the topic of a birthday party, which is familiar and relevant to her students and appears to connect with students' "real world" experiences of mathematics She also continues to encourage students' thoughts and responses which they share with her and the other students while the story is being read while the students collect information for their calendars, working with geometry "tubs" and writing about the photographs.

In Lesson G: *Geometry Tubs,* Lynne's attitude to exploration and discovery in mathematics is shown when she changes the labels on the tubs and then directs her students to "build something that moves". Polya defines such problem solving as "finding a way where no way is known" [from Krulik, 1980, p. 1]. Lynne's approach, in this example, also reflects

Frank's [1988] definition of problem solving as what you do when you do not know what to do.

Most of the students' mathematical exploration and discovery in Lynne's classroom is carried on in small cooperative problem solving groups. Lynne says that this type of teaching, where students are working in groups, frees her from the demands of managing and monitoring classroom behavior and enables her to spend considerable periods of time with individual students, giving assistance when it is really needed and helping them to reflect on what they are doing and seeing how they can extend their learning. She says that this way of grouping students allows her to spend time encouraging and helping students in their explorations and discoveries. The research of Noddings [1985] and Schrader [1985] suggests that the use of small cooperative groups to solve mathematical problems result in significant growth in problem solving competence. Research by Lochhead [1979], Whimby [1980], Schoenfeld [1983], Bellanca [1984] and Hoomes [1984] demonstrates that small group work facilitates growth in problem solving skills. Irons & Irons [1989] suggest that students' knowledge and excitement about mathematics increases if teachers provide experiences that allow discussion, an extension of known strategies and development of new techniques. During one of the problem solving work times it was observed that every student in Lynne's classroom was totally engaged in their mathematics work. Lynne commented that this was one of those “aha’s" of teaching!

Although Lynne's approach has characteristics which are supportive of mathematical inquiry, as discussed above, it was also observed that Lesson E: *Moira's Birthday* and Lesson F: *Birthday Calendars*, although they address topics that are familiar to Lynne's students - a characteristic

that would support mathematical inquiry - could also be interpreted as problems that focus on numbers, that are "number-oriented" [Garofalo [1993]. As in Lesson D: *Going to Camp,* such problems, often introduced by phrases such as "How many?" or "How much?" are referred to by Becker [1990, 1993]; Stonecipher [1986]; Polya [1963]; Bottge & Hasselbring [1993] as routine number-oriented or traditional problems. As in Lesson D: *Going to Camp,* such problems, according to Bottge & Hasselbring [1993] appear to connect with real life experiences but, in reality, seem artificially geared to specific operations and single, correct answers. In Lesson E: *Moira's Birthday*, Lynne asks "I wonder *how many* will come?" In Lesson F: *Birthday Calendars,* Lynne asks her students to find out *how many* days until their birthday. In asking these types of questions, Lynne's intentions could be interpreted as directing students to focus on the numbers in the problems. As discussed earlier in the chapter, Frank [1988] states that one of students' beliefs about mathematics is that mathematics is computation and that mathematical problems are easily solvable in just a few steps. Cobb [1984] states that students beliefs are developed from the perceived expectations of the teacher and the beliefs often do not reflect healthy and realistic views of mathematics. Lynne's presentation of problems that appear to focus on numbers, despite her intentions of encouraging students' exploration and discovery, could again reinforce students' beliefs that the goal of doing mathematics is to obtain the "right" answer [Frank, 1988]. This characteristic of asking "how many" might be problematic for mathematical inquiry.

In this second topic, characteristics of mathematical inquiry in

Lynne's classroom have been presented. Some of the characteristics of mathematical inquiry which have been presented are supportive of mathematical inquiry. Some of the characteristics presented have the potential to be problematic for mathematical inquiry. These characteristics are summarized in Figure 6 below and will be addressed further at the end of this chapter.

Figure 6: **Summary of Characteristics of Mathematical Inquiry in Lynne's Classroom**

Topic 2: Mathematical Exploration and Discovery

Lessons	Characteristics That Are Supportive Of Inquiry Orientation	Characteristics That May Be Problematic For Inquiry Orientation	Characteristics That Are Non-Supportive Of Inquiry Orientation
Lesson E ***Moira's Birthday***	Story and topic is familiar to students. Students present their plans to the class.	Lynne asks "I wonder *how many* more will come?" [a number-oriented question].	
Lesson F Birthday Calendars	Topic is familiar and interesting to the students. Lynne encourages students' talk about this learning experience.	Lynne asks the students to find out *how many* days until their birthdays [a number oriented task].	

Figure 6 [continued]: **Summary of Characteristics of Mathematical Inquiry in Lynne's Classroom**

Topic 2: Mathematical Exploration and Discovery

Lesson G Geometry "Tubs"	Lynne does not think that labeling the tubs was appropriate. Lynne organizes her students in problem solving groups. Lynne organizes the tubs so that students "build something that moves". Lynne indicates her excitement about watching students learn.		
Lesson H Photographs.	Students are invited to write about what they were doing in the photographs. Students confirm that they were interested in this learning experience because they had a part in the writing. Lynne comments that she loves to watch her students learn.		

Mathematical Discussion

The third way in which Lynne describes her inquiry-oriented approach to teaching and learning mathematics and mathematical problem solving is as an approach that encourages mathematical discussion. Lynne's way of working with students translates into teaching strategies for a

mathematics program in which most of the problem solving is done in small problem solving groups. Lynne feels that the advantage of this strategy for her students is that they can work in the group, discussing with each other and learning from each other’s efforts. Proudfit's research [1980] suggest that behaviors such as explaining the reason that a strategy is appropriate for a particular problem and explaining the implementation of a strategy are characteristics of successful problem solvers. One of the students working in these groups wrote in his mathematics journal about his experiences of working with geometrical forms in a mathematics "tub":

> I thought that this tub was very fun. I learned a lot of things and now I can show a lot of other people what I learned today. I chose to do these very shapes because I wanted to challenge myself. On the first one I had troble [sic] but the second one everybody in my group found it hard but not me. [Student's name] sort of helped me a lot.

Realizing that the strongest spur to actively formulating one’s own thoughts and feelings in order to communicate them to others, Lynne finds opportunities for one-to-one and small group interaction. Lynne says that encouraging her students to communally construct meanings allows them to internalize the process. She feels that this gives their constructions permanence. Lynne believes that when students are themselves the listeners and the occasional prompters, they are encouraged to think through the requirements of the task, using language to consider alternative courses of action and to evaluate their consequences before actually undertaking the activity. As Lynne says: "They do have answers if we let them talk and share their ideas".

Most of Lynne’s time is spent with individual students or in small groups, helping them to plan their activities and evaluate the outcomes.

The research of Proudfit [1980] suggests that evaluating strategies and solutions in light of the problems are characteristics that accompany successful problem solving. From time to time, Lynne brings the group together to clarify an area in which this group is experiencing difficulty. Often, students are willing to volunteer their suggestions or ask questions and so reveal to the teacher the framework they are using to interpret new information. Lynne's manner of interacting with the students is thus at the heart of her style of teaching, for it is this collaborative approach, a willingness to negotiate meaning, that encourages students to explore their understanding of a topic and gives them the confidence to try out their ideas without the fear of being wrong. Lynne encourages risk taking and sees that it is important in mathematics where errors as well as success can be productive. Lynne's encouragement of risk taking is supported by research [Polya, 1981; Silver, 1982; Good, Grows & Ebmeier, 1983; Lester, 1985; Kilpatrick, 1985; Becker & Shimada, 1993], which suggests that one of the insights into the process of problem solving, is for teachers to provide a safe, congenial environment for students to practice problem solving. The NCTM's *Standards* [1989] and *Professional Standards* [1991] state that one of the teacher's roles is to let students struggle with difficulties in mathematical problem solving.

Lynne models her way of experiencing mathematics by how she talks aloud with her students. Her teaching is most often done sitting with the students as they gather with her at the front of the room. With a free-standing "white board" beside her, she is able to talk about key points in the mathematics lesson and, even more importantly, her own thoughts on the subject by using such phrases as "I often think of" and "I wonder if". This way of working with students appears to bridge the gap between Lynne and

her students. When she says to her students "I never thought about it that way", her students see themselves as informing her understanding. Goos [1996] supports Lynne's modeling of mathematical thinking and states that when a teacher encourages such individual reflection, self-monitoring and checking, that teacher is moving students towards a culture of mathematical sense-making.

In classroom situations, Lynne sees discussion about mathematics as one in which she is often a learner along with her students as students offer differing perspectives on their mathematical work. The success of students' being heard and understood owes much to Lynne’s support and guidance. Her questions during mathematics help to maintain the focus of attention without imposing too tight a control over the direction it takes. Lynne says that she does not portray herself as the all-knowing person and her students recognize that "all the time". A real sense of partnership was observed in Lynne's classroom. Students are encouraged to contribute freely from their own experience. As they narrate their experiences to others, they are often discovering their significance for themselves.

It was observed that Lynne probes her students' thinking, encouraging them to explore their mathematical understandings at a deeper level. She does this often by using the phrase, "That's interesting. Can you tell me more?" in order to encourage students to extend their thinking about a mathematical concept. The *Professional Standards* [1991] states that such monitoring of and participation in students' discussions, as demonstrated by Lynne, are part of the role of the mathematics teacher and supportive of mathematical inquiry.

It was observed in Lynne's classroom that the students, in their mathematical discussions with their peers, often modeled Lynne's words as

they talked with each other. Such phrases spoken by the students and as contained in the data included: "Can you tell me about...., I think that......", and "What an interesting perspective"! Goos [1996] states that such cognitive scaffolding is particularly important for establishing a classroom community of practice.

Another strategy that Lynne used to encourage student discussion about their mathematical learning is called "Friday Files". This strategy entailed the students' use of file folders where they place what they consider very good or special work from any subject area, including most often, mathematics. The students took these folders home each Friday and shared their contents with their parents. Included in each file was a small booklet in which the students have written what they have learned and experienced during the past week and why they have chosen to bring home these pieces of work. It was observed that the students' comments in the booklets had been responded to by their parents. After they have been signed by the parents, these files are returned to Lynne on Monday and the routine continues into the next Friday. Lynne feels that Friday Files are a good evaluation strategy because parents are in touch with their child's learning and the children are given the opportunity to reflect and self evaluate on a weekly basis. Lynne recognizes that when children have something important to say to people who express interest, then the language and the thinking most fully interpenetrate as children struggle to capture what is observed and understood and to communicate that understanding to others. More importantly, Lynne commented, this applies in mathematics education which has been seen traditionally as less open to discussion.

Perhaps what is most striking about the observations of Lynne's

teaching strategies is the quality of her listening. When giving whole class instruction or working in small group or one on one, Lynne listens carefully to make sure that she understands her students' intentions. She will often repeat what the student has said in order to make certain that what she has heard is correct. Her questions and suggestions are couched in terms that students are able to understand and are designed to help extend their thinking. Even when she has to break off for a moment to respond to another student she keeps her attention around the student thereby signaling to her that this is only a temporary interruption. By listening attentively in this way, Lynne indicates that what students have to say is important, that they have expertise that is of value. When she asks questions it is in order to be better informed, not to check that the student's answer is in agreement with her knowledge about the subject. By listening to each student in this way, she builds up in each student a feeling of self-respect and confidence in what he or she knows and can do and, at the same time, a feeling of respect for others.

In this section, discussion has focused on the research findings related to mathematical discussion in Lynne's classroom. Lynne, in her classroom teaching of mathematical problem solving, provides opportunities for her students to discuss their learning of mathematics and mathematical problem solving by organizing them in small problem solving groups, by modeling mathematical talk and thinking, by providing opportunities for students to discuss their mathematics with their families and by the quality of her listening to students' mathematical discussions.

In this third topic, characteristics of mathematical inquiry in Lynne's classroom have been presented. As in the findings presented under the first

two topics in this chapter, some of the characteristics of mathematical inquiry which have been presented are supportive of mathematical inquiry. and some of the characteristics have the potential to be problematic for mathematical inquiry. These characteristics are summarized in Figure 7 on the following page and will be addressed further in the following section of this chapter.

Figure 7: **Summary of Characteristics of Mathematical Inquiry in Lynne's Classroom**

Topic 3: Mathematical Discussion

Lessons	Characteristics That Are Supportive Of Inquiry Orientation	Characteristics That May Be Problematic For Inquiry Orientation	Characteristics That Are Non-Supportive Of Inquiry Orientation
Mathematical Discussions	Students are organized into problem solving groups. Students work one to-one and in small groups. Lynne interacts collaboratively. Lynne encourages risk taking. Lynne models her way of experiencing mathematics. Lynne probes students' understanding. Lynne encourages her students to discuss their mathematics with their parents through "Friday Files". Lynne listens attentively to her students.	Lynne repeats what her students say.	

Summary

In the preceding sections of this chapter, the findings of the classroom data with reference to the nature of mathematical inquiry in Lynne's classroom have been presented and discussed. The findings were presented under topics: [1] Drawing On Students' Previous Learned Mathematical Attitudes, Skills And Thought Processes; [2] Mathematical Exploration And Discovery, and [3] Mathematical Discussion.

This concluding section of the chapter presents a summary of the findings, specifically those characteristics that support mathematical inquiry and those characteristics that have the potential to be problematic or non-supportive for mathematical inquiry.

The supportive characteristics include:

[1] Lynne chose topics for her students that appeared to be interesting and relevant to them.

[2] Lynne encouraged her students to share their comments, ideas and thoughts about their learning in mathematics.

[3] Lynne organized her students into problem solving groups and promoted a collaborative approach to learning.

[4] Lynne participated in her students' learning by circulating among them asking question, making observations, probing her students' understanding and listening to what her students were saying.

[5] Lynne encouraged her students to take risks.

[6] Lynne talked about not limiting her students' learning.

[7] Lynne said that she was excited about watching her students' learn.

[8] Lynne encouraged her students to extend their learning.

[9] Lynne allowed time for the exploration of mathematical concepts.

The analysis of the data from Lynne's classroom, as presented and discussed in this chapter also presented characteristics that had the potential to be problematic for mathematical inquiry. These potentially problematic characteristics include:

[1] Lynne modeled examples and strategies for her students that could be interpreted by the students as Lynne, the teacher, being the only source of mathematical truth;

[2] Lynne spoke of mathematical concepts such as numeration and mathematical operations in a way that may cause her students to interpret mathematics as a linear process.

[3] Lynne presented problems to her students that could be interpreted as number-oriented, routine and traditional exercises.

The analysis of the data from Lynne's classroom, as presented and discussed in this chapter presented characteristics that could be interpreted as non-supportive of inquiry orientation in her classroom. These non-supportive characteristics include:

[1] Lynne presented each student with a book of algorithms to practice numeration and mathematical operations.

[2] Lynne taught her students rules.

Overall, Lynne's approach to mathematics and mathematical problem solving sought to optimize the learning process by causing her students to become personally involved and to actively communicate, discuss and compare solutions, answers and approaches [Becker, 1992; Becker, 1993]. Essential to Lynne's approach is the development or selection of lessons that are conceived to provide an opportunity for any student, regardless of

ability, to find at least one solution and participate in the discussions. By creating more opportunities for her students, of all abilities, to actively participate in the classroom problem-solving process increases students' confidence in their ability to do mathematics [Nagasaki & Becker, 1993; Becker, 1993]. Lynne's students observing other students' discoveries or methods, comparing and evaluating students' different ideas and modifying their own ideas accordingly are important aspects of the inquiry approach. Such approaches to mathematics and mathematical problem solving include thought provoking questions, speculations, investigations and explorations [Tougaw, 1993]. Miller and Kandyl [1991] support Lynne's approach that students need the "knowing why" dimension of mathematical understanding in addition to the procedural understandings of mathematics. They define this "knowing why" as "implying an ability to recognize and to make use of a mathematical concept in a variety of settings, including some which are not immediately familiar" [p. 4]. The data suggests that the overall approach to teaching and learning is one of a less structured, more indirect style of teaching within a supportive classroom learning community.

However, in addition to characteristics that supported mathematical inquiry, the data also revealed characteristics that could be interpreted as having the potential to be problematic for mathematical inquiry. These characteristics, as discussed above, fall into an area of Lynne's teaching that, for purposes of this study, are referred to as "borderline" characteristics. "Borderline" means those characteristics of mathematical inquiry in Lynne's classroom that, from one perspective, appear to support mathematical inquiry, but, from another perspective do not. When considered in conjunction with students' beliefs about mathematics, as discussed in this chapter, these borderline characteristics could be

interpreted as problematic or, in some cases, as non-supportive of inquiry. One example is be Lynne's modeling of *Base Ten Blocks,* as presented and discussed earlier in this chapter. Lynne's modeling of *Base Ten Blocks* is seen as supportive of mathematical sense-making but can also be interpreted by the students as indicating that mathematics relies for its correctness on an expert [Lynne] and that mathematics has "only one way and only one answer".

This concept of "borderline" characteristics as introduced above is of interest and significance for this study because the classroom practice contained elements of both routine and non-routine problems. This will be discussed further in Chapter VI together with the findings from the problem-solving data which will now be presented in Chapter V.

CHAPTER V

THE PROBLEM SOLVING DATA
FINDINGS AND DISCUSSION

This chapter addresses the analysis of the problem-data. The purpose of this set of data was to explore and document the problem-solving practices of four students, chosen from Lynne's classroom, as they worked on routine, traditional mathematical problems.

As discussed in Chapter III, this second set of data for the study were collected from six audio taped problem solving sessions which extended over six weeks during May and June, 1994. Field notes were also taken during the audio taping sessions.

The analysis of the data was achieved by reviewing the transcribed talk of the four students in the research, as discussed in Chapter III. From the transcribed data of the students' talk throughout and upon completion of the data collection, events were elicited from which patterns in the problem-solving practices of the four students were identified. The significant events or, salient themes [Marshall & Rossman, 1995], were determined through a focused attention to the regularities or patterns in the data. Patton [1990] refers to these emerging events as important dimensions that are allowed to emerge from the cases under study. From these reviews of the data, five practices emerged as significant in addressing the guiding question. Of these five practices, four relate to mathematics and one relates to non-mathematical issues.

Each of the five practices will be discussed below, followed by examples from the transcript and a short summary. The chapter concludes

with a general discussion of the five practices.

Practices Brought Forth When Students, Learning Mathematics In An Inquiry-Oriented Approach, Are Presented With Routine, Traditional Mathematical Problems

Practice 1: Students pay immediate and significant attention to the numbers in the problems.

Throughout the transcript of the problem solving sessions, the data suggests that the students appear to pay immediate and significant attention to the numbers in the problems. In fifteen out of the seventeen assigned problems, the students focus immediately, within the first three minutes and 21 lines of the transcript, on the numbers in the problems. This focus continues throughout the problems. In the remaining two assigned problems, Problem #7 in Sudden Wealth and Problem #2 in Sudden Wealth, the students focus first on non-mathematical issues. However, after this initial focus on non-mathematical issues, the students, as in the other fifteen problems, focus on the numbers within the first five minutes and 29 lines of the transcript. This practice is significant because, as suggested by the data, in addition to the immediate attention to the numbers, the students rarely go beyond the numbers during their work on all the problems.

The following sections of the transcript from Problem 4, Problem 5 and Problem 6 of At the Movies provide an example of this first practice.

In these sections of the transcript it is also observed that the students start to solve Problem 4, then move on to Problem 5 and later to Problem

6. The movement from one problem to the others is smooth and goes unnoticed by the students for approximately fifteen minutes. Possibly, the students focus on the numbers of the problem so much, that they jump from one problem to another and even to a third, without apparently noticing what they have done.

In the transcript below, the students' references to the numbers in the problems are highlighted in order to emphasize that a significant part of the dialogue in this section of the transcript focuses on this practice.

In <u>At the Movies</u> Problem 4, Cristel starts by reading Problem 4 [lines 276 - 279].

Cr One evening the popcorn concession sold two hundred seventy five small boxes, one hundred eighty three medium boxes and plus fifty six large boxes. How much money did it take in?

The students attempt to solve this part of Problem 4 by multiplying the cost of a large box of popcorn [$2.25] by the number of boxes sold [56] [lines 280 - 284].

Li OK, large, large is **two twenty five**

Cr OK, so we need to do um

Li So **fifty six times two twenty**

Cr We need to do case

Ch What is it, **fifty six times what**

The students now list the numbers of boxes sold: 275 small boxes, 183 medium boxes and 56 large boxes [lines 285-289].

Li	OK, they, **two hundred and seventy** they sold **two hundred and seventy five** small boxes
Ka	Kay um **two hundred**
Li	A **hundred and eighty three** medium and **fifty six** large
Ch	OK so, OK so what is it, so **two hundred seventy five, two hundred seventy five**
Li	And **fifty six** large
Li	**One eighty, a hundred and eighty three** medium
Ch	A **hundred and eighty three, a hundred and eighty three**, and what was the last one
Li	And then **fif, fifty six** large
Ch	Now you have to add this all up? or times it all, add it right?

At this point, Lindsey skips to Problem 5 and introduces new information from this problem about the cost of cooking oil. One possible reason why she moves from Problem 4 to Problem 5 could be that the word "popcorn" appears in the first line of each problem as shown below:

Problem 4: One evening the <u>popcorn</u> concession sold...

Problem 5: The <u>popcorn</u> machine uses 12l of cooking...

Lindsey then suggests that the group multiply by fifty-two. This is information from Problem 5 [lines 300-304].

Li	You have how much does the, does the oil for **fifty two** weeks cost
Cr	So much money
Li	**Fifty four, fifty two** weeks, times **fifty two**
Ka	What

Lindsey now says that the group needs to add "this", possibly referring to the students' work from Problem 4 regarding their addition of the numbers of large, medium and small boxes [lines 305-308].

Li	We have to add this together and times it by **fifty two**
Ch	OK, I'm adding it
Li	OK
Ka	You're timesing it obviously

At this point in the transcript, it is interesting to note that Lindsey appears to jump to another problem, this time to Problem 6 [lines 331-333] as indicated by her reference to the final line of Problem 6, "How many boxes of popcorn did it sell?" A possible reason for Lindsey's second move could also be that the first line of the Problem 6 is similar to the first line of Problem 4 as shown below:

Problem 4: <u>One evening the popcorn concession</u> sold...

Problem 6: <u>One evening the popcorn concession</u> took in...

Li	All this **by fifty two** and then it says how much does it cost to um supply the popcorn at the movies

Christene reaches a total of "ten fourteen" to which Lindsey again indicates that they multiply by fifty two, referring back to the information from Problem 5 [lines 320-344].

Ch	OK, I've added it all and it equals **ten fourteen**
Li	OK
Ka	**Ten dollars and fourteen cents**

Cr	Here let me get this straight again kay
Ch	**Ten dollars fourteen cents**
Cr	**Seven five,** is it **two seventy five**. Wait, don't don't write anything
Li	**Ten dollars and fourteen cents**
Ch	OK, now we have to times that answer by what.
Cr	**Twenty five**
Li	**Fifty two**
Cr	Times
Ch	OK, **ten fourteen**
Cr	**Zero point seven**
Li	[unclear]
Ch	Times what Whoopsies
Li	**Fifty two**
Ch	**Ten fourteen times fifty two**
Cr	This is the answer
Li	Ya
Cr	So they got **two hundred**
Ch	That's the answer
Li	[noise]
Ch	That's the answer
	[laughter]
Cr	**Fifty two thousand seven hundred twenty eight**
Li	Dollars to supply popcorn, **fifty two weeks**, I'd think about it
Ch	That sort of, the answer is
Li	Think about it, **fifty two weeks**

Ch	**Fifty two,** is that all that it says

Li	Ya, oil

Ch	For **fifty two weeks**

It is not until this point, approximately fifteen minutes later and well into the problem, that Karen discovers that the group is on the wrong question. Interesting to note is that it is the number "fifty two" that appears to alert her to the fact that the group is on the wrong problem. Lindsey acknowledges this [lines 345-351].

Ka	Um for **fifty two** weeks, I don't see it here

Cr	Wha, what question are we doing kay, what question

Ch	We're not on that one, we're on this one

Ka	On this one

Li	Oh

Cr	Oh

Li	Oops

The students start Problem 4 again [lines 355-357].

Ka	One evening the popcorn session sold two hundred and seventy five small boxes, one hundred and eighty three medium boxes and fifty six large boxes of popcorn. How much money did it take in

The students are unsuccessful in solving the problem and abandon Problem 4 [line 409].

Ka:	Cause that was taking too long.

Some possible explanations have been presented above as to why Lindsey made the leap from one problem to another and then to a third These include: [1] the word "popcorn" appears in the first line of Problem 4 and Problem 5, [2] Lindsey uses the term "this" which possibly confused the students who thought she was referring to the numbers from Problem 4 when she had moved on to Problem 5, [3] The first five words of Problem 4 are the same as Problem 6.

A question around Lindsey's movement from one problem to another is why the other students followed her. Lindsey is identified and discussed in Chapter III as one of the better mathematics students in the group. Stacey's [1992] research on mathematical problem solving in groups suggests that students working in groups do not demand explanations from their peers about what they are doing. The data suggests that Christene, Cristel and Karen ask only for clarification of numbers or what operations they should do with those numbers rather than "demanding explanations". The routine nature of the problems on which these students are working could also be a contributing factor to the passive nature of the students in relation to these problems. Lochhead [1981], Heller & Hungate [1985] suggest that routine problems, such as those presented to these students and found in most textbooks, place students in the role of copiers where they are less active because they do not believe that there is any more for them to do. It would appear that these three students see Lindsey as a good mathematics student and follow her direction without questioning as she moves from one problem to another.

In conclusion, the sample data above suggests that the first thing that the students immediately do as they start to work on the problems is to focus on the numbers and this focus continues to receive a significant

emphasis throughout this thirty minute exchange. In addition, as noted in the sample data presented above, the students move from one problem to other problems without apparently noticing, possibly because of the significant attention they pay to the numbers in the problems. In the context of this data, this practice of paying immediate and significant attention to the numbers in the problem appears to be unsuccessful for the students as they did not solve Problem 4 nor did they return to it later.

Practice 2: The immediate and significant attention paid to numbers in the problems is always accompanied by an immediate focus on the calculations.

In all seventeen problems worked on by the students, once they focused on the numbers in the problems, this focus was accompanied within two minutes and ten lines of transcript by a focus on the calculations. This practice occurred even in those two problems when the students initially focused on non-mathematical issues. This focus on the non-mathematical issues was followed by the focus on the numbers and then, again, within two minutes and ten lines of transcript, accompanied by a focus on calculations. By calculations is meant the act of adding, subtracting, multiplying or dividing to find a result.

The following sections of the transcript from Carpet Sales provide an example of paying attention to the calculations of the problem.

In the transcript, the students read the problem and then, with Karen's suggestion, proceed to add the costs of the four types of carpeting. Interesting is the students' interpretation of the word "each" which becomes problematic. The students initially interpret "each" as adding all

the individual costs of the carpeting [lines 1406-1436]. Students' references to calculations, for example "plus" and "multiply" have been highlighted in order to demonstrate the students' attention to the calculations in the assigned problems.

Ka	Thirteen
Cr	Every, all the way through the house
Ka	Well, guys, guys, guys don't always, I knew this was going to happen again, don't, you have to explain stuff to us too
Cr	OK
Li	OK, so the harmony carpet is thirteen bucks right
Ka	Harmony carpet, where does it say that OK
Li	Right by the stupid guide [guy?]
Ka	Harmony carpet is thirteen dollars, **plus**
Ka	Kay the plush carpet
Ch	Plush carpet is twenty three
Ka	Twenty three
Li	Thirteen
Ch	**Plus**
Ka	**Plus**
Ka	The velvet carpet is nineteen
Li	**Plus** [unclear]
Ka	**Plus** the royal carpet which is twenty eight
Li	**Plus** [unclear]
Ka	Eight, eighty three dollars
Cr	No, but that's only
Li	Nineteen
Cr	What

Li	Eighty three
Ch	Eighty three
Cr	Ya. you guys, you guys,
Ch	Eighty three dollars
Cr	Scuse me
Li	What

Cristel states her interpretation of the word "each" ["count all these"]. Her interpretation of "each" would seem to reinforce how the group uses this word as <u>adding all the individual costs</u> of the carpeting. The students continue and indicate the next stage in solving the problem, which is to multiply the total cost of all the carpet by the number of tiles [sixty-one] in the floor plan of the house [lines 1437- 1441].

Cr	See um what you need to do is you need to **count all these**
Li	OK
Cr	And then, and then the um, **multiply them** by how many squares there are

Lindsey now suggests that they "plus it", that is the combined cost of the carpeting, "eighty three dollars" to the total number of tiles in the floor plan of the house [sixty-one] [lines 1442 - 1455].

Li	OK, so we've got the answer to this, now **we can just add, count these up and add it to this number**. OK
Cr	No, cause we have to **count the squares** [unclear]
Li	**Count up, count them** and then we can just **plus it** by eighty three
Ka	Kay, in the kitchen there's nine

Li OK

Cr Kay

Ch Kitchen there's nine

Li Kitchen there's nine

Ch So

[silence 20 seconds]

Ka Fifty five

Ch Yeah, fifty five

Karen multiplies eighty-three [the total cost of the carpets] by fifty-five [lines 1456 - 1457]

Ka Four thousand, five hundred and sixty uh, five

Ch Sixty five

Lindsey states her count of the total number of tiles in the floor plan of the house [line 1458].

Li We got sixty-one you guys

Christene adds eighty-three and fifty-five as one hundred and thirty-eight [line 1459].

Ch We got one three, I got one three eight

Lindsey states her count of the tiles as sixty five, this time [lines 1460 - 1463].

Li **Times** sixty five

Ch I got one three eight

Ka OK

Li Times sixty

Karen suggests that the group count the tiles again [lines 1464 - 1469].

Ka Kay, **let's count em again,** one, two,

Cr Just a minute

Ka **Three, four, five, six, seven, eight, nine, ten, eleven**

Ch **Two, three, four, five, six, seven, eight**

Li **Equals fifty one**

Cr No

Lindsey adds her count of the tiles, sixty-one, to the total count of the cost of the carpet, eighty-three, resulting in one hundred and forty four dollars [lines 1470-1487].

Li **Plus** sixty one. I got one hundred and forty four dollars [unclear]
[mumbling 13 seconds]

Ka Ya, I got sixty one now too

Cr Kay this is how much it would cost to carpet the whole house with harmony

Ka Hold on, just wait sixty one

Li Oh, OK thirteen sixty one

Ka **Plus** nineteen dollars

Ch **Plus** nineteen dollars

Ka **Plus ninety three dollars plus thirteen dollars plus twenty eight dollars, a hundred and forty four**

Li Uh

Cr This is what I wrote kay

Ch One hundred and forty four dollars

Cr **I counted these up**, so if you um if you did the whole house with harmony carpet

Cristel, who made the initial reading of "each" now appears to question the adding of each type of carpet and multiplying this total to the number of tiles. Did the group misinterpret her when she said "you need to count all these"? Cristel now appears to indicate that she means to multiply the cost of each type of carpet by the number of tiles. It is difficult to tell from the transcript if she had been indeed continuing with her own calculations as the others worked on their own interpretation of her words [lines 1488-1494].

Ka I'm going to do it once more

Cr It'll be seven hundred ninety three dollars

Li Kay, so we know it's sixty one now **we just have to times it with all these** kay

Cr Sixty one

Li And the price, the price with sixty one

Cr **I timesed it with that**

The rest of the students continue to add the number of tiles with the total costs of all the carpets despite Cristel's suggestion. Cristel is silent for much of the following dialogue [lines 1495-1521].

Ka I got a hu, I still got a hundred and forty four

Cr No but

Ch Same

Cr No, but here, let me, let me try this here

Li	Nineteen
Cr	What you need to do is **you need to go**
Li	**Nineteen plus**
Cr	Thirteen Sixty one
Cr	Three point zero zero, eighty times
Ka	How'd you get that
Cr	Sixty one [unclear]
Ka	Oh, cause it's dollars
Li	Duh
Ch	I know
Ka	But you're only, [student's name], you're only [unclear]
Li	[unclear] velvet [unclear]
Ka	You're only um doing this one
Ch	Ya, ya you're only doing the harmony carpet
Ka	You've got to do all four
Ch	You have to do all four
Ka	And it, it costs a lot more than seven hundred and ninety three bucks
Ch	Ya
Ch	**Because twenty eight plus nineteen plus thirteen plus thirteen would be** [unclear]
Cr	That's how much it would be, that's how much it would be

In conclusion, the sample of the transcript presented above suggests that students, as they pay immediate and significant attention to the numbers in the assigned problems, also pay immediate attention to the

calculations in the mathematical problems presented to them. In reference to this practice and Practice 1, Krutetski [1976] observed that students who over rely on number considerations put much more of their time and energy into activities related to calculation. These efforts include checking computations excessively.

The students were unsuccessful in solving Problem 1 of Carpet Sale. The students spent large portions of their problem solving time on answer giving activities such as attention to numbers and calculations without thinking about the nature of the givens and goals and producing inferior solutions. The students in this study spent a large portion of their time, approximately 80% of the 45 - 60 minute problem solving sessions, working on numbers and calculations.

Practice 3: Students focus on key words in the problems.

In addition to the practices of paying attention to the numbers and calculations in problems, as discussed above, the students in the research pay attention to certain words which, for them, act as a "key" to the answer to the problem. Although not as significant as the attention paid to the numbers and the calculations, the students' focus on key words appears frequently in the data.

Sections from the transcript of Problem 1 and Problem 2, of Carpet Sale are now presented as an example of this practice. In the following sections of the transcript, the students appear to "attach" themselves to the word "estimate", and "actual" which they use frequently. The students possibly see that, if they can figure out what to do with these words, they can solve the problem.

This section of the transcript also suggests the struggle that the

students experience with the mathematical concept of estimation.

In the following section of the transcript, students' references to "estimate" and "actual" are highlighted in order to indicate the students' attention to these words.

In Carpet Sale, Christene states that the problem requires the students to estimate. [lines 1396 - 1405]

Ka	The floor plan shows the main floor of a house each small square represents one square meter
Li	Ooh small house yup
Ka	[unclear] twenty number 1
Ch	**Estimate** the cost of the carpeting the main floor using each kind of carpet
Li	The harmony carpet's thirteen
Cr	Kay what are we su
Li	Harmony carpet thirteen dollars
Li	**Estimate** cost of carpeting using

The students continue to work on the problem until Cristel refers to estimation in line 1528. She states her interpretation of "estimate" as "you're not supposed to find it out" [lines 1528 - 1530].

Cr	**It's estimate, you're not supposed to find it out**
Ch	I know that
Li	**It's estimate** down here

It is difficult to tell from the transcript what Lindsey means by "It's estimate down here". However, it is important to note that, in this problem, the students do not appear to formally estimate.

As the students move to Problem 2, the group again talks about estimation. Karen reads and interprets the problem.

Ka		Kay, how much would it cost to use Velvet carpet in each room, show how you got your **estimate** [lines 1684-1685].

Ka		Now we've just gotta **estimate** how much the kitchen then [line 1705].

Lindsey states her interpretation [lines 1711-1712].

Li		Now we can't do the **estimate,** you just **estimate** the three

Karen states her interpretation of the problem. She introduces the word "actual", possibly her interpretation of the first sentence of Problem 2, "How much would it cost to use Velvet carpet in each room?" Her words "it says" seems to confirm for her that the problem asks for the actual cost first [lines 1713 - 1717].

Ka		OK, [student's name] **actual cost** it says, let's put it, OK one of the kitchen, nine

Li		But we have to **estimate** first

Ka		Well why don't we do the **actual cost first**, that's a lot easier

The text of the problem implies that the students should use estimation before calculating the actual cost. The data suggests that the students are confused about whether they are being asked to find the actual cost or the estimate. During this work on Problem 2, Karen uses the word "actual" and the others appear to imitate her. Karen seems to use the word "actual"

to differentiate from "estimate".

The students now continue to interpret estimation [lines 1718 - 1727].

Li	Because, then our **estimate** we'll just put an **estimate** like one dollar under
Ka	Fine
Li	We can't do that
	[laughter]
Ka	Put it twenty bucks under
Ch	Ya
Ka	Easy, cinch
Li	Uh
Cr	I'm doing the **estimate** first
Li	I'm doing **estimate** first you guys

Christene appears to imitate Karen's use of "actual". Karen appears to use the word "real" as a synonym for "actual" [lines 1754-1755, 1819-1821, 1825-1827].

Ka	Kay, now the **actual cost**
Ch	The **actual cost**

Ch	The the, **actual,** the es, the **actual**, no the **actual cost**
Ka	No, no, oh yeah that's the **actual cost** No
Ch	Ya, because the **actual cost,** cause we took all the carpet

Ch	We did this the the **estimate** first
Ka	Oh no that's not wha, that, kay, that was, that's the **real**

number.

In this section of the transcript, the data suggests that students pay attention to key words, in this case "estimate" and "actual" which seem for them to be a key to solving the problem. In addition, the data suggests that the students appear to struggle with what is expected of them in the problems and with the concept of estimation. In the context of this discussion of students paying attention to key words, it is important to note that the students did not solve Problem 1 nor Problem 2 of Carpet Sale.

Practice 4: Students look back to the problem statement only to see if the numbers are copied right.

This fourth practice elicited from the data is that of students looking back to the problem statement to see if the numbers are copied right. This practice, as the others, is contained throughout the data. Problem 2 #6 of Sudden Wealth is presented and discussed below to provide an example of this practice. Relevant sections of the transcript are highlighted in order to indicate this practice of looking back to check the numbers [lines 1054-1083].

Li	How much tax do you have to pay
Ka	Plus ninety five cents?
Ch	**Plus, no plus zero point sixty four**
Li	Kay, point sixty four
Ka	Zero point sixty four
Li	Times seven four six
Ch	You've got all mixed up
Ka	OK let me try it again

Cr	**OK, start over**
Ka	OK what is it
Li	It'll cost four hundred and seventy seven dollars and forty four cents for just the tax?
Ka	**Eight**
Ch	**Four eighty five**
Ka	**Eight four**
Ch	**No, eight no eight OK clear, eight**
Ka	**Eight?**
Ch	Dot ninety
Ka	**Ninety five right?**
Ch	**Ninety five**
Li	**How much tax you have to pay for the whole school so that isn't right [name of student]**
Ch	**Plus zero point sixty four**
Li	**This isn't, how much tax, we have to pay for the whole school**
Ch	**Sixty four times seven four six equals**
Li	**So just sixty sixty four times seven four six**
Ka	Ya
Cr	**Sixty four**
Ch	Got it?

The discussion of this practice of looking back only to check the numbers in problems is informed by the studies of problem solving procedures. From the four-step heuristic model of Polya [1945] and subsequent studies, there is an indication that one of the processes that

accompany successful problem solving is the strategy of "looking back". Proudfit [1980], in her research of problem-solving processes of fifth grade children, writes that students who evaluate the solution of a problem in light of the problem conditions are likely to be more successful in mathematical problem solving. This "looking back" is to determine whether the plan for solving the problem is correctly carried out. In the sections of the transcript above, there appears to be little indication of this "looking back" as reviewing the work in an organized way. The data suggests that the students look back only to check that they have the right numbers.

Practice 5: Students are concerned about non-mathematical issues.

Throughout the transcript, the data suggests that the students demonstrate pre-occupations with non-mathematical issues. These issues include the students' concerns about performance and their place within the group. Each of these two concerns will be discussed below.

Concerns About Performance

Throughout the transcript, there is a sense of the students rushing to complete the problems. This concern about performance is interpreted as getting the problems done and possibly getting them done first. As an example of this concern about performance, the final lines of each of the seven problems of <u>At the Movies</u> are presented below. These examples from the data suggest a sense of concern about getting the problems "over with".

In Problem 1, the students end the problem by simply stating the numbers, most often, in the form of money [lines 198-103].

Li	That's easy, four dollars and eighty cents
Ka	Ya so he had four dollars and eighty cents left
Li	Ya
Ch	I'll write this down
Li	Four dollars and eighty cents
Ka	Four eighty

In Problem 2, the students end the problem again by simply stating the numbers in the form of money and talking about who is doing the reading and writing [lines 163-174].

Li	So four
Ch	Four dollars five cents read the next question
Ka	Hey, I'd like to write some stuff here too
Li	OK, so four point
Ch	OK, I'll read the next question OK what where is,
	Actually, we've read two so you guys get to read one each now
Cr	I've already read one
Li	How much money did he spend
Ch	OK, I'll read this one
Li	Hey, hey, I haven't read one yet
Ch	OK, you read the number three

In Problem 3, the students again address the numbers and again talk about who is doing the writing [lines 264-274].

Li	OK, so it's twenty

Cr Twenty dollars and fifty cents

Ka That's what I have at my original one but then I didn't think that was right so I erased it

Ch Well you were right, I get to put this one down

Ka No, I do I do

Li [unclear]

Cr [unclear]

Ka You're always writing on this

Cr OK

Ch I don't care

In Problem 4, the students decide to abandon the problem because it is taking too long. The students state that they will return to the problem later. However, they do not return to this problem. Note that the students do not state that they will skip this problem possibly because they don't believe they have this choice given the research context [lines 402-409].

Li Twenty six thou, twenty, twenty six thousand, one hundred and twenty five

Ch Let's let's go on to a different one

Li OK, we'll just circle this

Ka Cause that was taking too long

In Problem 5, the students end the problem by stating the money. Karen refers to what the amount of money means personally to her [lines 432-440].

Ch That's the answer? Kay, let's put down eighty eight

	dollars
Ka	That was sort of an easy one
Li	OK, Oops, wrong thing
Ch	Kay, how's eighty eight zero zero, hey, they got a lot of money
Li	Oh ya
	[laughter]
Ka	I have more than that

In Problem 6, the students end the problem by talking about the numbers, who will write the answer and about the light on the taping equipment [lines 572-579].

Ch	You write it down, you write it down, write the answer
Ka	Oh
Ch	Kay, what's the answer
Cr	You were supposed to, OK, OK, OK
Li	Five hundred and seven ya
Ka	Ya
Cr	Ya
Ch	Ya

In Problem 7, the students end the problem by stating the numbers and making HELLO on the calculator [lines 780-802].

Li	I got a hundred eighty nine for the medium soft drinks
Ka	I got I can't I don't even know how much that, I think I did mine wrong
Ch	OK

Cr	OK, and how much did you do for this
Li	I didn't do that one yet
Cr	Kay, do the seventy seven one
Li	OK
Ch	I spelled hello, H E L L O
Li	OK seventy, seventy seven times
Ch	H E L L O
Li	How much are the large
Ch	I did, I spelled hello
Li	Large
Ch	Hey, [student's name] you wanna see how you spell hello
Li	Large
	[laughter]
Ch	Wanna see how you do it, I mean
Ka	No guys, guys
Li	Oh crap, I missed that
	[laughter]
Ch	Nice, I hope the whole class hears that
	[laughter]

Additional examples of final lines from other problems are presented below to indicate that students' concerns about performance occurs throughout the transcript.

In Sudden Wealth Problem 1 #1, students end the problem by naming the number and talking about who writes the answer [lines 827-831].

Ch	I write the answer down

Cr A hundred eighty

Ch No, whoever figures it out gets to write it down

Ka I did

Ch So you get to write down the answer

In Sudden Wealth Problem 3, students discuss how they will work together [lines 1382-1383].

Ch Let's work together and they work together you get all the answers I don't care.

In Carpet Sale Problem 2, the students end the problem by talking about their answers [lines 1932-1934].

Ka Cause that's what I had too

Ka [Student's name] said it wasn't right

Ch That's it

From the sections of the transcript presented above, as well as throughout the transcript, the data suggests that the students are concerned with performance and with getting the problems done.

In the following sections of the transcript, it appears that the questions the students ask during their work on the problems are ones regarding performance issues such as clarification of numbers, procedures for calculations and what the group is doing. The data suggests that these questions, asked by the students, indicate a sense of speeding up the problem solving process and "getting the work done".

Karen asks for clarification about the cost of a movie ticket [lines 32-35]

Ka What's three seventy five?

Li Oh three seventy five

Cr Point three five

Li Three point seventy seventy five

Christene asks about procedures for solving the problem [line 68].

Ch No, don't you ya, you add this all together?

Cristel asks about the group,

Cr What are you doing, what are you doing, what one? [line 1869]

It is Lindsey who occasionally asks questions for her own learning [lines 138-140, 252-253].

Ka Twenty eight, uh, twenty eight dollars and sixty two cents

Li So that's how much he spends?

Ch And then there's three seventy five

Li Why don't you just use the times sign?

Concerns About Place In The Group

In addition to the non-mathematical issue of concern about performance, the data suggest that students in the problem solving sessions are concerned with the non-mathematical issue of their place within the problem solving group.

Karen best demonstrates this preoccupation with her place within the

group. The section of the transcript below suggests this practice as Karen talks about being ahead of or behind the other three students.

Karen states the numbers and her being ahead of the others [line 885-886].

Ka	We've already done that, you guys are behind it's seventy two dollars

She discusses her concern about being behind [line 1031].

Ka	Hold on guys, just wait, just wait

Ka	Guys you guys are going ahead of us [line 1101].

Karen states her reasons for not wanting to be behind [lines 1157-1160].

Ka	You guys you guys are way ahead of us I think
Cr	But we're just trying to
Ka	Ya guys it's not fair to us though cause you guys are gonna figure out all the answers

Ka	Guys this is so confusing every one of us kay
Cr	Got a different answer
Ka	We've all got different answers and you guys are rushing ahead of us and it's not fair if you guys figure out all the answers and we just copy you [lines 1179- 1183]

Ka	Ya but you guys are going ahead of us, you guys got to help us, you don't just go ahead [lines 1639-1640]

Karen has been described in Chapter III as being very social with her classmates. The research of McDonald [1990] about the role of cooperative learning in mathematics suggests that classmates in a mathematics class expect to help each other and that the amount and type of this interaction is largely determined by the students' personality, actual mathematical ability and self-perceived mathematical ability. Karen's pre-occupation with her place in the group reflects her perceived social status and perceived mathematical ability.

In the samples of the transcript presented above, the data suggests that the students indicate their concern about what Overson [1989] refers to as getting mathematics "over with". They demonstrate this through ending their work on the problems by stating the numbers, talking about who is doing the reading and writing, abandoning one problem, commenting about the taping equipment and about making words with the calculators. In addition, the questions that the students ask, as presented above, also suggest that their concerns are about performance. It appears that the students see themselves as successful when they complete the mathematical tasks quickly.

This section has also presented an example of the non-mathematical issue of place within the problem solving group.

Summary

For these students, attempting to solve the mathematical problems presented to them in the study was a long and arduous task The data suggests that the students valiantly worked through all the problems, using a variety of practices evoked by the context and the problems themselves.

The five practices, as elicited from the data, were mostly ineffectual in their quest for the solutions to the problems. These practices, four of which are mathematical in nature and one of which is non-mathematical are:

Practice 1: Students pay immediate and significant attention to the numbers in the problems.

Practice 2: The immediate and significant attention paid to numbers in the problems is always accompanied by an immediate focus on the calculations.

Practice 3: Students focus on key words in the problems.

Practice 4: Students look back to the problem statement only to see if the numbers are copied right.

Practice 5: Students are concerned about non-mathematical issues.

Regarding the first four practices of the students which are mathematical in nature, the data suggests that these students demonstrate practices that are commonly associated with traditional number-oriented mathematical problem solvers.

In his study of number-oriented problem solvers, Garafalo [1993] discusses certain number consideration strategies used by students while solving routine problems. Like the practices used by the students in this study, these strategies include focusing on numbers to decide which operations to use in order to solve a problem, to assess one's progress and

to adjust one's plans. These are regarded as number consideration strategies because, in using them, these students often paid more attention to the numbers present in the mathematical problems or resulting from carried out operations than to either the quantities represented by these numbers or to any relationships between these quantities. According to Garafalo, the students in this study who focused on these number considerations often read the problem for just the numbers and the key words, looked back to the problem statement to see if the numbers are copied right and paid much attention to calculation, often more than to problem understanding. Garafalo says that these students seem to view mathematical problem solving as something to get "over with" in almost any way they can. Skemp [1987] addresses this type of mathematical problem solving as "rules without reasons"; that is, using mathematical rules without understanding why or where these rules came from.

The Curriculum and Evaluation Standards for School Mathematics [1989] states that symbol manipulation and computational rules do not contribute to important aspects of children's mathematical development. These ineffectual practices include finding exact forms of answers, attention to reading, writing and ordering numbers symbolically and the use of clue words to determine which numbers to use.

In relation to the fifth practice of the students, which is non-mathematical in nature, Overson's [1989] research discusses students' self-preoccupation with non-mathematical tasks, which includes their concerns about performance. As mentioned previously, Overson refers to this as "doing mathematics". These non-mathematical practices can be non-cognitive in nature but they have the capacity to enhance or interfere with the cognitive and metacognitive resources brought to the problem solving

context. Goos [1996] states that students' perceived order in the group inhibit effective problem solving. Cobb [1986] states that for number-oriented problem solvers, like the four students in this study, the goals often appear to be more social in nature than mathematical.

In conclusion, the data from this study suggest the power of these routine, traditional problems to evoke ineffectual strategies. This power to bring forth such practices, the nature of mathematical inquiry in the classroom and the relationship between these two sets of data will now be discussed in Chapter VI.

CHAPTER VI

DISCUSSION AND SUMMARY

This sixth chapter consists of an overview of the study and a summary of the findings. Also included in this chapter is a discussion of the findings, conclusions and implications of the study.

Overview of the Study

The purpose of this study was to address the tension between the NCTM's major focus on problem solving and teachers' struggles in developing mathematically meaningful problem situations [Nagasaki & Hashimoto, 1984]. As discussed in Chapter I, the *Standards* [NCTM 1989] state that if a mathematics program is to be consistent with its goals, objectives, mathematical content, and topics the mathematical problem solving approaches, materials and activities should be compatible with its vision and intent [*Standards* 1989, p. 241].

In order to address the guiding question of this study, two sets of data were collected. The purpose of these data was to: [1] document the nature of mathematical inquiry in the selected elementary classroom and [2] explore the mathematical problem solving practices of four students from this classroom when they were presented with routine, traditional problems.

Summary of Findings

The guiding question of this study was: What mathematical practices are elicited when four Grade III/IV students, learning mathematics in what is described by their teacher as an inquiry-oriented

approach, are presented with routine, traditional mathematical problems? A summary of the findings in response to this question will be presented under two headings: [1] The Nature of Mathematical Inquiry in Lynne's Classroom and [2] Routine, Traditional Problems and the Problem Solving Practices Of Four Students.

The Nature of Mathematical Inquiry in Lynne's Classroom

Lynne described her approach to mathematics and mathematical problem solving as inquiry-oriented. Her description, as described by Tougaw [1993] and the "vision" of the NCTM Standards [1989] for the teaching and learning of mathematics and mathematical problem solving suggested three topics for the exploration and documentation of Lynne's inquiry orientation: [1] drawing on students' previous learned mathematical attitudes, skills and thought processes; [2] mathematical exploration and discovery, and [3] mathematical discussion The nature of mathematical inquiry in Lynne's classroom was presented and discussed in Chapter IV.

The analysis of the data from Lynne's classroom, as discussed in Chapter IV, presented three sets of characteristics of Lynne's teaching practice in relation to mathematical inquiry which will now be presented, followed by a discussion.

One set of characteristics presented were those characteristics that were supportive of mathematical inquiry. These supportive characteristics included:

[1] Lynne chose topics for her students that appeared to be interesting and relevant to them.

[2] Lynne encouraged her students to share their comments, ideas and

thoughts about their learning in mathematics.

[3] Lynne organized her students into problem solving groups and promoted a collaborative approach to learning.

[4] Lynne participated in her students' learning by circulating among them asking questions, making observations, probing her students' understanding and listening to what her students were saying.

[5] Lynne encouraged her students to take risks.

[6] Lynne talked about not limiting her students' learning.

[7] Lynne said that she was excited about watching her students' learn.

[8] Lynne encouraged her students to extend their learning.

[9] Lynne allowed time for the exploration of mathematical concepts.

The analysis of the data from Lynne's classroom, as presented and discussed in Chapter IV, also presented characteristics of her teaching practice that had the potential to be problematic for mathematical inquiry. These characteristics included:

[1] Lynne modeled examples and strategies for her students that could be interpreted by them as Lynne being the only source of mathematical truth;

[2] Lynne spoke of mathematical concepts such as numeration and mathematical operations in a way that could cause her students to interpret mathematics as a linear process.

[3] Lynne presented problems to her students that could be interpreted as number-oriented, routine and traditional.

The analysis of the data from Lynne's classroom, as presented and discussed in Chapter IV also presented characteristics of her practice that could be interpreted as non-supportive of inquiry orientation. These

characteristics included:

[1] Lynne presented each student with a book of algorithms to practice numeration and mathematical operations.

[2] Lynne taught her students rules for learning mathematical computations.

Discussion of the Three Sets of Practices

Lynne organized her problem solving lessons around topics familiar topics to her students, such as their coming to school and going on field trips. Her lessons on geometric shapes and number concepts were organized in such a way that her students had the opportunity to draw on events and mathematical concepts that were familiar to them in an environment that encouraged students to share their thoughts and explanations in small group settings.

The problem solving lessons that Lynne presented to her students provided opportunities for them to explore and discover mathematical concepts of number through reading stories and the use of calendars on which children were invited to note the birthdays of their classmates. Exploration and discovery of mathematical concepts were also encouraged through problem solving lessons where students, in small groups, were encouraged to extend, solve and pose problems about number concepts. Students were also encouraged to talk about their learning experiences.

Lynne encouraged mathematical discussion through the organization of her students into small problem solving groups. Lynne also modeled mathematical talking and listening, often becoming a learner with her students as she questioned and wondered aloud about the concepts under discussion.

Evident in the description of Lynne's classroom was that Lynne's approach to mathematical learning and teaching and mathematical problem solving was supported by the school learning community. The approach to teaching and learning that was occurring in Lynne's classroom was supported by a conscious effort of other teachers, administrators and parents to build a school environment which supported such responsible and independent learning. This effort was reflected in the physical and organizational design of the school and in the support of the parents, administrators and visitors at Hilltop School.

In addition to these characteristics of Lynne's practice that supported mathematical inquiry, the data also revealed characteristics of her practice that had the potential to be problematic or even non-supportive.

The potentially problematic characteristics of Lynne's teaching are referred to in this study as borderline characteristics of her practice. As introduced in Chapter IV, borderline characteristics represent those characteristics of Lynne's teaching that, from one perspective, appear to support mathematical inquiry, but from another perspective, particularly when considered in conjunction with students' beliefs about mathematics, could be interpreted as problematic for or non-supportive of inquiry.

The concept of borderline characteristics of Lynne's practice is important for this study because it is in this area that the potential lies for the overlap of Lynne's mathematical inquiry with more traditional approaches to the teaching of mathematics and mathematical problem solving. Although Lynne's approach appeared inquiry-oriented, she presented problems to her students that were routine problems, similar to the routine problems presented to the students in the problem solving sessions of this research study. The two sets of problems were similar in

that they asked questions such as "How many?" or "How much?" and focused on numbers and calculations. Further discussion of Lynne's presentation of routine problems in her classroom will take place following the summary of the second set of data below.

Routine, Traditional Problems and the Problem Solving Practices Of Four Students

When the students, selected from Lynne's classroom, were taken to a section of the multi-purpose room and presented with three sets of routine problems, they demonstrated five problem solving practices. These practices, as summarized below, focus almost exclusively on the numbers, calculations and key words in the problems. Non-mathematical practices were also discussed. These practices, for the most part, proved unsatisfactory for the students' attempts to solve the problems.

Practice 1: Students pay immediate and significant attention to the numbers in the problems.

In the seventeen problems presented to the students, the students' focus is significant and sustained on the numbers in the problems. The focus is significant because, in fifteen of the seventeen problems attempted by the students, they turned to the numbers in the problems almost immediately. The attention to the numbers is also significant because the students seldom, if ever, go beyond the numbers in the problems. Examples provided in Chapter V include one example in which three students follow Lindsey from one problem to another without apparently noticing they are on different problems. It is Karen who eventually notices the group is on a different problem. Of interest is that she notices this move from one

problem to another by identifying an unfamiliar number in the problem.

Practice 2: The immediate and significant attention paid to numbers in the problems is always accompanied by an immediate focus on the calculations.

In fifteen of the seventeen problem presented to the four students, the focus on the numbers is followed closely by a focus on calculations. Even in the two problems where the significant focus on the numbers occur slightly later in the problem, this focus on the numbers is followed closely by a focus on the calculations. As Krutetski [1976] indicates, students who rely on number considerations put excessive amounts of time into calculations. It appears that, after the students turn immediately to the numbers in the problems, they turn to the calculations in order to "do something" with the numbers. As Frank [1988] suggests, one of the beliefs that students hold about mathematics is that all mathematical problems are solved by the application of mechanical procedures. This would appear to be the case for these students and the problems presented to them.

Practice 3: Students focus on key words in the problems.

In the examples provided in Chapter IV, students focus on the key words " estimate" and "actual". Similar to Practices 1 and 2, the focus paid to these words appears excessive, almost as if the students regard such words as a "key" to their problem solving. As Stonecipher [1986] states, poor problem solvers focus on the surface details of a problem.

Practice 4: Students look back to the problem statement only to see if the numbers are copied correctly.

As the research of Proudfit [1980], Duncan [1985] Stonecipher [1986]

Driscoll [1983] suggest, good problem solvers look back through their problem to check the reasonableness of their answers. The students in this study show little evidence of looking back over the problems for this reason. They appear to look back only to check if the numbers have been correctly copied. As Frank [1988] and Garofalo [1989] indicate, one of the beliefs that many students hold about mathematics is that there is no need for them to evaluate their answer.

Practice 5: Students are concerned about non-mathematical issues.

In the problem solving sessions, students are concerned about two non-mathematical issues. One of these issues is about completing the assigned problems as quickly as possible. Throughout the problem solving sessions, there is a sense of rushing through each set of problems in order to finish the problems and possibly to complete them first. As Frank [1988] suggests, when students are faced with a traditional problem solving task, one of the approaches that they take is to attempt the problem as if it were a textbook exercise and seek a rule or procedure that leads to a quick answer.

Another non-mathematical issue is the students' concerns about their place in the problem solving group. Karen appears to be the student who is most concerned about this issue. She is also the one who Lynne describes as very social with her classmates. Karen herself also states that she did not like mathematics and likes to work in these problem solving sessions because of the social aspects. There appears to be a relationship between Karen's perception of her social status in the groups and her place in the group during the problem solving sessions. The research of McDonald [1990] would appear to support this non-mathematical issue. Further discussion of these problem solving practices will take place in the

following section of the chapter.

Discussion

This discussion addresses the findings of the problem solving data in relation to the findings of the classroom data. The discussion will attempt to provide possible explanations for the students' problem solving practices in relation to Lynne's struggles in developing mathematically meaningful problem situations.

In this study, Lynne represents "good" mathematics teachers whose practices reveal multiple layers of teaching strategies, beliefs and mathematical tasks as they face the implementation of a mathematics program with a very different philosophical base from the previous, more traditional program. This new and different philosophical base constitutes an approach to mathematics where, as the literature indicates, mathematics can be seen as a language, as a kind of reasonable structure, as a collection of knowledge about numbers and space, as an arrangement of methods for deriving conclusions, as the core of the understanding of the physical world or as an engaging intellectual activity [Davis & Hersh, 1981; Kitcher, 1988; Lakotas, 1976; Tymoczko, 1986].

In Chapter IV, students' beliefs about mathematics were presented as a factor in their problem solving practices. Embedded in the students' beliefs is an older, more traditional view of mathematics. The literature describes this view as one in which mathematics is seen as a cold and austere discipline which provides little scope for judgment or creativity. Such beliefs, shared by students and by society, include: [1] mathematics is computation and doing mathematics means memorization and following rules; [2] mathematics problems are quickly solvable in just a few steps; [3]

the goal of doing mathematics is to obtain the "right answers"; [4] the role of the students is to receive the mathematics and demonstrate this reception of information on the next test and [5] the role of the teacher is to transmit the mathematical knowledge and to verify that students have received this knowledge [Fey, 1979; Confrey & Lanier, 1980; Buerk, 1982; Schoenfeld, 1983; Silver, 1982; Carpenter et al, 1983; Wheatley, 1984; Confrey, 1984; Cobb, 1984 and Frank [1988]. These beliefs are seen in this study as significant in their effect on the mathematical teaching practices of Lynne and other teachers like her.

The discussion that follows is organized in relation to the characteristics of Lynne's practice, as presented in Chapter IV and summarized in this chapter. In the first section, discussion will focus on those characteristics of Lynne's practice described in this study as borderline characteristics. These characteristics are discussed first because they include Lynne's teaching practice of presenting of routine problems. This practice is significant because it addresses the purpose of this study, teachers' struggles in developing meaningful problems for their students.

Following the presentation of Lynne's borderline characteristics, those characteristics that appear to be supportive of mathematical inquiry are presented. Finally, the non-supportive characteristics are briefly addressed, followed by a concluding summary.

Borderline Characteristics of Lynne's Teaching Practice and Students' Problem Solving Practices

This section of the chapter attempts to relate the borderline characteristics of Lynne's teaching practices to the problem solving practices of her students. Discussion of the borderline characteristics will

address Lynne's modeling of strategies, her presenting of mathematics as a linear process and her presenting of routine problems to her students.

The first borderline characteristic of Lynne's practice is her modeling of examples and strategies such as "tallies and graphs" [Lesson A] and addition with *Base Ten Blocks* [Lesson B]. Goos [1996] states that introducing such tools for mathematical communication is an important aspect of the teacher's role for establishing a mathematical community. Lynne also models her way of experiencing mathematics by how she talks with her students about her own thoughts on the subject. Goos [1996] supports such modeling of mathematical thinking and states that when a teacher encourages such individual reflection, self-monitoring and checking, that teacher is moving students towards a culture of mathematical sense-making.

From another perspective, Lynne's modeling could be interpreted by her students as Lynne being the only source of mathematical correctness. There is literature that supports this perspective that, when teachers model mathematical strategies and operations, students may interpret the modeling by the teacher as the only way to solve the problem correctly [Frank, 1988; Garofalo, 1989]. An example of this perspective of Lynne's modeling is seen in Lesson A [Who Walks to School?] and Lesson B [Addition Using *Base Ten Blocks*].

In relation to this characteristic of Lynne's "modeling to correctness", it appears that Lynne models examples and strategies more often when the content of what she is teaching is mathematical computation. An example of this is in Lessons A and B. This raises the question whether such lessons containing numbers and computation also elicit Lynne's more traditional teaching practices such as Lynne modeling to her students how to reach a

correct answer. This question will be addressed throughout this discussion in relation to other characteristics of Lynne's practice and the topic of computation.

The Third International Mathematics and Science Study Videotape Classroom Study [1999], referred to in this study as the TIMSS [1999], offers an insight into Lynne's practice of modeling. The study suggests one principle that many teachers share which underlies their decision to teach in a certain way is "the best way to learn something is to acquire it through a clear, orderly, incremental process". The TIMSS [1999] suggests that, if teachers adhere to such a principle, then procedures for problem solving would be clearly demonstrated so students would not flounder or struggle" [p. 137]. Lynne's modeling in Lessons A and B might reflect her belief that lessons with numbers and computations require this clear, orderly approach with less potential for making errors.

Lynne's teaching objectives for Lesson A are the data management strategies of "tallies and graphs" and for Lesson B, addition with *Base Ten Blocks.* Both of these mathematical strategies are contained in the Grade 3 and 4 provincial and national curricula. Lynne's setting of curricular objectives for her lessons and her modeling of strategies could provide the message to her students that she is the only model of why certain problems are chosen and of how these problems should be solved.

The belief that mathematical problems do not arise from real situations but are imposed by a curriculum with pre-determined objectives is in contrast to what Garofalo [1993] describes as the approach used by meaning-oriented problem solvers. Such problem solvers are encouraged to try to form meaningful interpretations of the conditions and quantities in the problems, for example, how to record the ways in which students come

to school and how to manage such data and then use their own understandings to develop meaningful plans of action in order to solve the problem. Such an approach to the problems would possibly result in many different mathematical processes and results by the students.

This part of the discussion has focused on Lynne's modeling and possible reasons for such practices. The following discussion addresses how Lynne's modeling is a possible factor in the problem solving practices of her students.

Lynne's practice of modeling the correct way to an answer would appear to be a factor in the students' traditional, approach to the solving of routine problems.

The following two approaches to mathematical problem solving reflect Lynne's modeling in Lessons A and B and the students' practices in the problem solving sessions. In both classroom Lessons A and B and the three sets of routine problems attempted by the students, the approaches to mathematical problem solving appear to be similar to the Dahmus' [1970] "memorization" approach. This is described as a technique for problem solving where an algorithm is developed that will handle a class of problems. Students are trained to identify and implement the appropriate algorithm. In classroom Lesson A, Lynne identifies and implements the algorithm "tallies and graphs". In Lesson B, she identifies and implements procedures of addition with the *Base Ten Blocks*. In the routine problem solving sessions, the students choose the appropriate algorithm which is applied immediately after the numbers are found. Once the algorithms are discovered by the students, they appear to understand that the use of these algorithms will lead them to the correct answer.

The approach to problem solving used in classroom Lessons A and B

and in the problem solving session is also similar to one described by Covington & Crutchfield [1985] as an "imitation" approach, where students are trained to model a master problem solver. In the classroom Lesson A and Lesson B, the students possibly place Lynne in the role of the master problem solver. In the problem solving sessions, it appears that the students attempt to place one of their peers in a similar role. The students, without Lynne as their master problem solver, appear to turn to Lindsey as their model. Lindsey has been identified as one of the better mathematics students in the group. She is also the oldest and, physically, the largest student in the group. Lindsey presented herself during the problem solving sessions as a "wise sage" During the sessions, she was often silent, sitting back, giving out advice and illuminating inconsistencies in the calculations. It was Lindsey who led the students from Problem 4, to Problem 6 [<u>At the Movies</u>] without any of them noticing that they had made this move. All four students continued to work on what they thought was Problem 4. Although the meaningless nature of the problems has been discussed as appearing to be a factor in the passive nature of the students' problem solving, another possible factor could be that the students have simply replaced the classroom teacher, Lynne, with Lindsey as their problem solving model. Lindsey is seen by her three peers as a competent mathematics student. She is older and bigger than the rest and could be perceived as a suitable problem solving model.

In summary, Lynne's practice of modeling strategies and algorithms, along with her students' possible interpretation of her role as a master problem solver, could be factors and, possibly, explanations for some of their problem solving practices in which they identified and implemented what they considered appropriate algorithms.

The second borderline characteristic of Lynne's practice is her presenting of numeration and mathematical operations in a way that could be interpreted by her students as a linear approach to mathematics.

Lynne claims to be an inquiry-oriented mathematics teacher when she describes her approach as one that encourages her students to use their previously held understandings in order to explore and construct mathematical understandings and make sense of their learning. However, Lynne also says that she needs to find out early in the year what her students already know about numeration. She also says that if her students don't get the numeration, they won't get the big number systems. It appears that Lynne's rhetoric reverts to more traditional ways of talking about her students' mathematical understandings when the focus of her teaching is on certain types of mathematics, such as Lesson B, which is on computation. Nagazaki & Hashimoto [1984] state that teachers often struggle with this lack of pedagogy. This raises questions about whether teachers feel comfortable in allowing students to construct their own mathematical understandings, particularly in relation to computation skills. Do teachers have a pedagogy for this new approach or do they feel compelled to "give students the skills" [i.e. numeration skills] in order to make certain that students can attain, as Lynne states, the "big number systems"?

This tension between "inquiry" talk and "traditional" talk is also revealed in how Lynne presents numeration and mathematical operations as a linear process. When she teaches the four mathematical operations of addition, subtraction, multiplication and division, she presents them in a sequence, suggesting an understanding that each of these operations build on each other and, therefore, must be taught in pre-determined sequence.

When teachers present each of these operations, they are most often accompanied by mini-lessons, practice booklets and review quizzes. Lynne was observed teaching mathematical rules such as one to help her students to memorize the nine times table.

Despite claims to an inquiry approach, Lynne's practices reveal a layer of her teaching that appears to indicate her understanding that mathematics concerning numeration and operations is a linear process rather than a process of exploration and construction. The TIMSS [1999] indicates that teachers, like Lynne, who believe that they are addressing current NCTM reforms in mathematics, also believe that mathematics is useful as a set of skills. The TIMSS [1999] study suggests that such beliefs would be reflected in lessons, such as Lynne's, that revolve around the practice of those skills.

Garofalo [1989] states that the nature of the classroom environment in which mathematics is taught strongly influences how students view the subject of mathematics, the way mathematics should be done and what students perceive as the appropriate response to mathematics problems. Students, when presented with a mathematics program as a set of skills and reinforced by their own and society's traditional beliefs about mathematics, are possibly being given the message that, as the literature indicates, mathematics is computation and doing mathematics means memorization and following rules

This message that mathematics is a linear process appears to be part of the mathematical understandings of the students as they worked on the routine problems in the problem solving sessions. The nature of the problems themselves, described by Frank [1988] as a collection of exercises solved quickly using learned facts, rules and procedures has been discussed

as a possible contributing factor to the routine problem solving practices of her students. Throughout the problem solving sessions, the four students, given the routine nature of the problems, first searched for the numbers and then applied the calculations. According to Frank [1988], when faced with a problem solving task, students in the traditional classroom adopt one of three approaches: The first is not to accept the problems as true mathematics and refuse to attempt or to make a weak attempt at the solution. The second is to attempt the problem as if it were a textbook exercise and seek a rule or procedure leading to the quick answer. The third is to try to employ a general problem solving strategy for a short period of time. Frank [1988], found that these approaches appear to contradict the problem solving process. The students, when faced with a traditional classroom view of problem solving attempted the problems as if they were textbook exercises. The students sought out a rule or procedure which they hoped would lead to a quick answer. This approach was reinforced by the nature of the problems and the students' own beliefs about mathematics.

In summary, Lynne's practice of presenting numeration and mathematical operations as a linear process could be a possible factor in the students' attending so quickly and significantly to the surface features of the problems. A reason why teachers like Lynne persist in this traditional practice may be their lack of pedagogy for dealing with mathematical problems, particularly problems that contain numeration.

The third borderline characteristic of Lynne's practice is her presenting of problems that could be interpreted as number-oriented, routine problems. This practice raises significant questions about why inquiry-oriented teachers like Lynne persist in presenting such routine,

traditional problems to their students.

In Lessons A to H, Lynne provides a broad variety of problems to her students. These problems range from "open-ended" to those that are defined by Bottge & Hasselbring [1993] as routine, traditional problems. An example of an "open-ended" problem is Lesson H: Geometry Tubs in which Lynne asks her students to "go build something that moves". Lessons D and E are examples of routine problems which ask such questions as "How much?" and "How many?" These problems are similar to the routine problems presented in the problem solving sessions which also begin with "How much?" and "How many?" and are geared to specific operations and concrete answers.

Lessons D and E, despite Lynne's intentions to provide an inquiry-oriented approach, appear as traditional, routine problems, "dressed up" in her attempt to address students' interests. She attempts to make connections with such "real life" student experiences as going to camp and birthday parties. Her choice of topics of interest to her students, according to Becker [1992, 1993], appear to support mathematical inquiry.

The problems presented to the students in the problem solving sessions also attempted to connect with "real life" through scenarios about money in connection with going to the movies, becoming suddenly wealthy and planning a home. Bottge and Hasselbring [1993] state that such connections as attempted by Lynne in Lessons D and E and presented in the problem solving sessions, appear to students to be on the surface of , rather than arising from real situations. Bottge and Hasselbring [1993] also state that students do not associate these problems with their own experiences because they perceive such problems as making artificial "real world" connections.

Lynne's presentation of routine problems to her students raises questions about why she felt that such problems were legitimate and appropriate. Did Lynne realize that the problems presented to her students were routine? Did she possibly think that, because the problems addressed her students' interests, they were legitimate for her inquiry approach? What embedded beliefs led her to offer these exercises? Was it the power of the computations that led her to present such problems? The "power of computation" has been discussed in a previous section of this chapter about Lynne's modeling strategies and examples in lessons with computations and her approach to lessons of numeration and operations.

Another question that arises is why mathematical lessons, such as geometry Lessons C and G, do not appear to have the same power to bring forth more traditional approaches to mathematics as the lessons with computations? Do teachers such as Lynne feel compelled to focus on computations and routine problems because computation is regarded in society's traditional beliefs about mathematics, as, possibly, being "more important" than geometry? Lynne's practices of modeling the correct way to solve problems containing computation, her linear approach to mathematical processes such as numeration and her presenting of routine problems appear to indicate a possible understanding that students must "get the computational part of mathematics right" in order to address these beliefs that "doing mathematics" primarily involves "doing computation".

This discussion can be extended into issues of teachers' accountability to the educational stakeholders. As discussed earlier, Lynne and teachers like her are seen as "good math teachers". To the stakeholders, this often means primarily "good teachers of mathematical computation". Teachers believe that correct computational skills are concrete evidence of their

students' progress.

Geometry, on the other hand, has traditionally been seen by teachers and, possibly by society, as "less important" than the learning and practicing of calculations in elementary school mathematics. Teachers will often leave geometry and other "less-computational" topics to be taught "if there is enough time". Given the less important status of such mathematical topics as geometry in mathematics classrooms, there is less pressure on teachers like Lynne to "get the geometry right". It is possible that, when teachers perceive that they are under less pressure of accountability, they show a more inquiry-oriented layer of their teaching.

This leads to a more general discussion on Lynne's practice in the context of the "power of computation". Lynne's modeling of strategies occurs most often in her lessons which focus on computation, such as Lessons A, B and E, and less, if at all, in her geometry lessons. In the geometry lessons, such as Lessons C and G, she appears to be more relaxed in her control of the teaching situation. In geometry Lesson C, she moves back from the centre of the teaching circle and allows her students to take more ownership for the discussion which evolves into student comments such as: "the world is a sphere" and "the classroom is a square". However, in her classroom lessons on computation, Lynne often appears at the front of the class, as in Lessons A where she models strategies, or at the overhead projector modeling examples of addition, as in Lesson B, or reading about a birthday party to her students, as in Lesson E. In these three lessons about computation, Lynne appears to feel more compelled to place herself in a central position with respect to her students, a position where she possibly feels more in charge and leaves little room for error in order to "get the computation right".

Another question arises in considering teachers' understandings about their teaching approaches to computation as compared to other topics such as geometry. When presenting Lessons E [*Moira's Birthday*] and F [Birthday Calendars] which focus on computation, Lynne provides the number-oriented questions for her students [i.e. How many?], rather than allowing her students to bring forth their own questions about the birthday story and the calendar activity. However, in Lesson G [Geometry Tubs], Lynne allows her students to explore freely and ask their own questions when she invited her students to "build something that moves" Why such a different, less constrained approach? Was Lynne concerned that, if she did not ask the questions about *Moira's Birthday* and Birthday Calendars that her students might not ask the "right questions"? Would Lynne possibly think that by allowing students to ask their own questions, she risks the possibility that she would not be able to complete her objectives for the lessons?

These issues of control in lessons on computation call for some hypothetical explanations for why Lynne presents routine problems to her students. Lynne adopts an approach to teaching numbers and calculations that is more traditional and routine. She does this because, like other good mathematics teachers, she possibly lacks a pedagogy for developing mathematically meaningful problem situations. This traditional approach may be in response to what teachers perceive as the demands and constraints placed on them by the curriculum, by educational stakeholders and by students. These people see mathematics as primarily computation and success in mathematics as success in computation. Lynne's teaching is nested in traditional beliefs of educators, students, and society about mathematics.

Working within these constructs, how do teachers like Lynne, continue to see themselves as inquiry-oriented teachers? The TIMSS [1999] offers some insights. In the TIMSS [1999], when asked if they were implementing reform in their mathematics classroom, most American mathematics teachers in the study believed, like Lynne, that they were implementing such reforms. When asked how they judged if they were in accord, a vast number of the teachers stated such surface features as their use of manipulatives and cooperative groups and "real world" connections as indicative of their alignment with current reform in mathematics. These features are similar to Lynne's teaching practices which also include manipulatives such as *Base Ten Blocks* [Lesson B], "real-life" scenarios such as Lesson A: Who Walks to School? and group work. Lynne, like the teachers in the TIMSS [1999], possibly believes that, since she uses such teaching strategies, she is an inquiry oriented teacher. It would appear that using such surface features allows teachers to present themselves as "inquiry-oriented", and like Lynne, they truly believe this, as indicated by their rhetoric. These teachers present their inquiry orientation even as they present "mathematics as computation" in order to address issues of accountability.

In summary to this discussion about Lynne's presentation of routine problems to her students, some possible reasons why teachers like Lynne continue to present routine mathematical tasks to their students include the persistent belief that "doing mathematics" is primarily "doing computation" and that computation is considered by teachers to be concrete evidence of their students' progress or lack of progress. Also, presenting routine problems to her students possibly reinforced her students' traditional beliefs about the nature of mathematical problems and the ways in which they are

to be solved. They simply regarded such problems in the classroom and in the sessions as routine exercises designed to practice computation. Discussion has also been presented in this section about teachers' understandings of mathematics problem solving involving computation and the relationship of these understandings to teaching constraints posed by the curriculum and educational stakeholders.

In this section on the borderline characteristics of Lynne's teaching practice and the students' problem solving practices, Lynne has been presented as representative of those teachers who attempt to incorporate current changes in the teaching and learning of mathematics and mathematical problem solving.

Lynne's practices of modeling mathematical strategies to one answer, presenting mathematics as a linear process and presenting routine problems are possible explanations for her students' problem solving practices. The students also modeled to one answer and attended to the problems as a linear process.

Other factors influencing the problem solving practices include the nature of the problems presented to the students as well as the traditional beliefs of the students about mathematics and mathematical problem solving.

In relation to teachers presenting routine problems to their students, discussion in this section has focused on hypothetical reasons why teachers persist in this practice. These possible reasons include teachers' understandings that surface features such as "real word" connections identify them as inquiry-oriented teachers despite the routine nature of the problems they present. Teachers may believe that, because problems contain "real world" connections, they are legitimate for inquiry-oriented

classrooms and are aligned with the NCTM. Another possible reason why teachers persist in presenting routine problems to their students is the embedded societal beliefs about computation and mathematical success.

Supportive Characteristics of Lynne's Teaching Practice and Students' Problem Solving Practices

This section of the chapter attempts to relate the classroom data and those characteristics of Lynne's classroom practice that support mathematical inquiry to the students' problem solving practices in the data. This discussion also continues to explore the multiple layers of Lynne's practice and the practices of teachers like her, in an attempt to provide the relationships between their teaching and the problem solving practices of their students. The discussion will be organized around the nine supportive characteristics, as presented in Chapter IV.

The first supportive characteristic of Lynne's practice is her choice of topics interesting and relevant for her students. Choosing topics of interest for students has been identified as being an important skill for teachers [NCTM, 1989; NCTM, 1991]. However, despite the appeal that such topics might have for Lynne, it is questionable whether students see all problems presented in classrooms as relevant for them. Lynne chose the topics of coming to school, birthday parties and going to camp. In the TIMSS [1999], teachers who cite as one feature of good instruction the use of real-world problems, do implement such features in their lessons. However, according to the TIMSS [1999], this feature alone does not necessarily indicate the quality of the instruction as intended by the NCTM Standards [1989] and, in fact, may only bear a superficial relationship to the quality of instruction. As the TIMSS [1999] states: "Quality of

instruction depends on how features are implemented" [p. 126].

This study suggests that students had very little input into the choice of topics for the classroom lessons. The question arises did Lynne assume that the topics she chose would be of interest to all her students? Did she consider that some of her students might have negative experiences of birthday parties or going to camp?

As the literature suggests with little input into the problems, Lynne's students may see them as traditional text book exercises, artificially presenting topics on the surface, rather than arising out of real situations [Bottge and Hasselbring, 1993]. The problems in both cases provide little choice but to be treated like any other traditional exercises designed for computational practice.

The second, third and fourth characteristics supportive of inquiry in Lynne's practice are her encouragement of students to share their comments, ideas and thoughts about their learning in mathematics; organization of her students into problem solving groups in order to promote this collaborative approach to learning; and her participation in her students' learning by circulating among them asking questions, making observations, probing her students' understanding and listening to what her students have to say. Lynne says that "They do have the answers if we let them talk and share their ideas". These types of teaching practices are regarded in the literature as important in promoting problem solving competence [Noddings, 1985; Schrader, 1985; Lochhead, 1979; Whimby, 1980; Schoenfeld, 1983; Bellanca, 1984; Hoomes, 1984; Irons & Irons, 1989; Becker, 1992, 1993; Becker & Shimada, 1993; Artz, 1996; Goos, 1996]. Irons & Irons [1989] suggest that students' knowledge and excitement about mathematics increases if teachers provide experiences that

allow discussion.

There is evidence in this study that Lynne structured the group work to maximize the chances that her students would, as the literature suggests, engage in questioning, elaboration, explanation and other verbalization in which they could express their ideas and through which the group members could give and receive feedback [Artz, 1996]. However, the problem solving data revealed that the only group work practices demonstrated by students were concerns about performance and place in the group. There is little or no evidence that the students in the problem solving sessions shared their comments, ideas and thoughts about their learning in mathematics in the meaningful way that Lynne had encouraged them to do.

Statements from the students, as presented in Chapter III, provide some insight into why students did not work together in such a meaningful way. The students discuss what they consider the benefits of group work for them. Their reasons, as the literature suggests, often appear to be more social than mathematical [Cobb, 1986]. The students' comments include: "four heads are better than one", "we all put our answers together to see which ones seem most appropriate", "we learned how to wait for others and how to talk about what they were doing", "the group made it fun" and "talking to the others in the group about their answers". These comments of the students reveal that, in groups, as Artz [1996] suggests, students may share answers, do one another's work or they may help one another, with a multitude of possible communication patterns in between. According to Artz [1996], these behaviors, similar to the behaviors of performance and place in the group from the problem solving sessions, indicate that Lynne's attempts at structuring within a group do not appear to have been successful.

In summary to this discussion on the second, third and fourth characteristics, Lynne's teaching practice of grouping her students and providing guidance for structuring the problem solving groups appears to support problem solving. However, despite her attempts at providing this guidance for the classroom group work, the students in the problem solving sessions focus on performance and place, revealing little of Lynne's practice. The students appear to fall back on their beliefs that, as Garofalo [1993] suggests, the goal of problem solving is to satisfy the teacher and to avoid failure. This could possibly explain why the four students focus so much on performance and their place in the group during the problem solving sessions rather than adopting the meaningful strategies that Lynne had encouraged in her classroom.

The sixth, seventh, eighth and ninth supportive characteristics are Lynne's encouragement of her students to take risks and explore mathematical learning. These characteristics have been discussed as supportive of mathematical inquiry. In her classroom, Lynne provided opportunities for students to take risks and provided examples of lessons that appeared to place few limits on her students' learning. Lesson G: *Geometry Tubs* provided such an example when Lynne said that she took the labels off the geometry tubs because "that's limiting kids". She also said "the kids know what they can do. When you limit what students can learn, they will not try".

Lynne's excitement about her student's learning was woven throughout the classroom data. Her excitement about learning was reflected in the positive atmosphere in her classroom and in her students. Throughout this study, in the classroom and in the problem solving sessions, the students remained open to the task and persisted with whatever

was presented to them!

In the previous section, the observation that Lynne's approach to teaching geometry appeared to be less "controlling" and more inquiry-oriented than her teaching of lessons involving computation was discussed. Another observation concerning "control", is the time allotted to lessons pertaining to computation. The lessons which addressed computation [A, B, D, E and F] were, for the most part, completed during one daily mathematics period, rarely if ever going past that time. However, the time allotted by Lynne for Lessons C [Geometry Tubs], and G [Photographs], extended into work either at home or in another classroom time period This raises possible questions about how Lynne allotted time to certain mathematical tasks. Teachers like Lynne consider computation important and, thus, "needing to be done right" for students' success. These teachers organize computational tasks so they are completed in one mathematics period and under the teacher's supervision. Another factor for consideration is that students possibly see these number-oriented tasks as ones to be completed as quickly as possible under the teacher's supervision. This partially explains the issues of performance and position in the group during the problem solving sessions.

The NCTM [1989] suggests that students need to work on problems that may take hours, days and even weeks to solve in order to learn to value mathematics, become confident in their abilities to do mathematics, become mathematical problem solvers and learn to communicate mathematically and learn to reason mathematically [p. 5]. A hypothetical question is how teachers, parents and students would respond to this new approach of "taking time to work through problems" in contrast to traditional approaches of "just get it done"?

The following summary to this section continues to explore the multi-layered nature of Lynne's practice. This re-visitation has revealed that those layers of Lynne's practice that were initially described in this study as supportive of mathematical inquiry appear to be less supportive when examined in more depth.

According to researchers, the use of an inquiry-oriented approach involves a diversity and variety of problem solving activities, including providing students with an opportunity to solve unfamiliar problems. This approach seeks to optimize the learning process by inviting the students to become personally involved and actively communicate, discuss and compare solutions, answers and approaches [Becker, 1992, 1993]. Lynne's approach appeared to invite her students to become involved but it appears questionable that her students felt personally involved and engaged. To what extent did Lynne's approach encourage her students to actively and meaningfully communicate in their problem solving groups?

Although Lynne presents topics intended to be of interest for her students, the students have limited input into the choice and personal involvement and, nevertheless, possibly perceive these problems as routine. Lynne grouped her students in the classroom and encouraged meaningful mathematical interaction; however, the students' personal comments about group work and their interactions in the problem solving sessions and did not reflect the meaningful communication that Lynne promoted.

Lynne encouraged her students to take risks, yet, the students in the problem solving sessions very seldom, if ever, went outside of the rules of mathematics and adopted what is, according to Garofalo [1993], a number-oriented problem solving style. Although Lynne attempted not to limit her students' learning and encouraged exploration of mathematical learning, at

times her students' learning appeared to be constrained by the amount of time she allotted for particular activities.

Previous discussions have indicated that Lynne's teaching practices in computation often appear as different from her practices in other areas of mathematics, such as geometry. These teacher practices could possibly explain why the students in the problem solving sessions adopted the approach that mathematics problems are quickly solvable in just a few steps, as the literature implies. Given the continuum of mathematical understandings inherent in the classroom teaching practices, students possibly may not perceive anything but traditional mathematics teaching, particularly when the practices reflect what they and society believe mathematics to be.

Non- Supportive Characteristics of Lynne's Teaching Practice and Students' Problem Solving Practices

In this section, a brief discussion will be presented on those characteristics of Lynne's practice that appeared to be non-supportive of mathematical inquiry and the relationship of these teacher practices to the practices of the students in the problem solving sessions. These characteristics are briefly discussed because they appear more obvious than the other sets of characteristics in relation to their non-support of mathematical inquiry and their possible effect on the students' problem solving practices in the sessions.

The two non-supportive characteristics of Lynne's practice are her presenting each student with a book of algorithms to practice numeration and mathematical operations, and her teaching of rules to her students.

These teaching practices appear to provide the clearest message for

the students in the classroom and the problem solving sessions that, as the literature suggests, mathematics is computation, memorization and following the rules to obtain the "right" answers.

Why do teachers like Lynne persist in such traditional practices? Previous discussions have provided some hypothetical reasons. The data in this section of the study again suggests that non-supportive characteristics of Lynne's practice arise in relation to issues of computation. One reason for this may be that inquiry-oriented teachers teach in a more traditional way when the mathematics being taught contains computation. Teachers possibly do this in order to address issues of accountability. Teachers could understand that "doing mathematics is doing computation". Such beliefs appear to translate into mathematical practices that place a certain status on ways of teaching computation compared to other mathematical topics.

This section of the study has addressed the relationship between what and how students learn mathematics and mathematical problem solving. Some possible explanations for the problem solving practices of the students in the problem solving sessions are presented. Lynne, like virtually all strongly inquiry-oriented teachers, has multi-layered instructional patterns, some supportive of mathematical inquiry, some not.

Lesson A : Who Walks to School? possibly provides an example of Lynne's multi-layered instructional patterns. In this lesson, Lynne attempts to provide a topic that addresses students' interests and arises from a "real life" situation. On one level, Lynne talks about her own situation and how she comes to school, models the "tallies and graphs" strategies and allows for exchange among her students as well as for class discussion and presentation. She allows her students to synthesize and reflect on their

learning experiences. These teaching practices would appear to support an inquiry orientation to mathematics. On a closer glance, other layers of Lynne's practice in this same lesson presents her as a master problem solver for her students, modeling one method of solving a problem for the right answer. Additionally, she presents practice booklets for her students. These factors and layers of Lynne's teaching practice provide important messages for students about what it means to "do mathematics", messages that are also personal and individual for each student as they, themselves, bring additional layers to the learning task.

CHAPTER VII

CONCLUSIONS AND IMPLICATIONS

The purpose of this study was to address concerns in relation to the tension between the NCTM's major focus on problem solving and teachers' efforts to develop mathematically meaningful problem situations [Nagasaki & Hashimoto, 1984]. The study concludes that Lynne endeavors to develop meaningful problem situations. Her teaching presents a continuum of mathematical practices, ranging from practices that are supportive of mathematical inquiry to those that are potentially problematic, questionable and even non-supportive of an inquiry orientation.

Concerning the guiding question of this study, the study concludes that the students in the problem solving sessions, despite learning in what their teacher describes as an inquiry-oriented approach, when presented with routine problems used mathematical practices that were also routine and traditional. The four number-oriented practices and one non-mathematical practice demonstrated by the students proved ineffective for solving the problems.

This study concludes that the students' problem solving was affected by a variety of factors, including the nature of the routine problems, traditional, embedded beliefs about mathematics and the nature of teachers' mathematical inquiry in the classroom.

The study suggested that Lynne's approach to mathematics presented many and varied opportunities for her students to engage in problem solving tasks and activities. The study also concluded that Lynne, in these efforts to present meaningful problem solving situations for her students

continued to present routine problems. Reasons for Lynne presenting such problems to her students are suggested in terms of understandings of mathematical teaching and learning and mathematical problem solving.

One understanding is that teachers have themselves experienced mathematics as a given, as acquiring facts and memorizing rules. Now these teachers are being asked to invent a kind of teaching that engages students in complex reasoning and in authentic contexts. Ball [1996] writes that teachers are faced with trying to find ways to connect students with mathematics and mathematical reasoning and to engage students in genuine experiences despite the fact that they have never seen or experienced such teaching. Teachers, like Lynne, think that, because some problems are interesting to students, they are appropriate for an inquiry approach.

For teachers, like Lynne, a paradox emerges. As an elementary teacher, she is the product of the very system she is trying to reform. This system promotes mathematics as a set of procedures focusing on observable behaviors, not on mathematical thinking. Battista [1994] writes that this system views the role of the mathematics teacher as progressing through carefully scripted schedules of skill acquisition. Teaching mathematics, according to this system, means also telling students how to perform procedures. In addition, these teachers understand their role to be one that must ensure students' success in completing the mathematical tasks given to them. In order to ensure this success, teachers attempt to reduce the mathematical tasks to rigid step-by-step procedures. Teachers reason that, if students follow these steps, they cannot fail.

Another reason for her practice of presenting routine problems may be that, as a product of the "pre-reform" system, Lynne may have limited confidence in her mathematical knowledge. When Lynne asks her students

"How much?" or "How many?", she may be guiding them along a path where she feels confident about her mathematical knowledge. If she left such questions open to her students, she risks venturing into unknown territory. Selecting an "open-ended" problem or task for her students, would require Lynne to "see" the mathematics latent in its scope. Moving in this direction could be possibly intimidating, even treacherous, for teachers like Lynne when they are unsure of the terrain being explored. Presenting "safe" problems and ways of solving these problems keeps risk of failure to a minimum for teachers and students.

In summary, the major findings from this study are:

[1] It appears that teachers like Lynne present teaching practices in mathematics and mathematical problem solving that are multi-layered in nature. This study suggests that Lynne and teachers like her come to the NCTM "vision" eager to explore its possibilities but with deeply, embedded traditional beliefs, attitudes and skills about mathematics, mathematics teaching and mathematical knowledge. With eagerness and good intentions, teachers like Lynne "attach" practices such as the use of manipulatives, "real-world" connections and group work onto their established teaching strategies. They adopt these new and often surface strategies without understanding fully the deeper issues that accompany them. Teachers adapt these strategies to their current teacher practices and to what they already know about mathematics teaching and learning. This may explain why, as teachers endeavor to implement an inquiry-oriented approach, they continue to present traditional mathematical tasks.

[2] This study concludes that teachers may be struggling to adapt to the relatively recent, quick and radical changes in philosophy and practice

regarding the NCTM vision. On the basis of the data, this study concludes that such a shift has been possibly more complex for teachers and students than has been realized. Teachers are expected to change their practice and the way they work with students, moving from "talk and chalk" to "guide on the side". Outside factors of accountability, such as Standardized Tests, affect teachers' change in addressing the NCTM "vision" for mathematics and mathematical problem solving.

[3] Both the classroom data and the problem solving in this study indicate the depth of teachers' and students' embedded beliefs and the significant effect of such beliefs on the teaching and learning of mathematics and mathematical problem solving. The study concludes that good mathematics teachers, like Lynne, talk the NCTM rhetoric along with more inquiry-oriented practices. These beliefs and practices affect students' learning and the NCTM vision of providing equal opportunities for all children to become lifelong learners, informed citizens and mathematically literate consumers and workers [Standards, 1989].

If we consider the depth of the changes and the effect of societal embedded beliefs, then the education of teachers in mathematical problem solving appears to be insufficient. The implications of this study are addressed in this section in relation to teacher professional development and future research in mathematics education.

Ambitious efforts are underway to reform mathematics curriculum and instruction in mathematics. This study suggests that we take a closer look at what we think we know about teacher learning and the teaching envisioned by the NCTM reforms. This study reveals that the NCTM reforms challenge culturally embedded views of mathematics held by

teachers like Lynne. Realizing the NCTM "vision" will require intensive societal and individual learning - and unlearning, not just by teachers but by all stakeholders.

This study has revealed that teachers like Lynne present some practices that are supportive of inquiry and the NCTM "vision" and some that are potentially problematic and non-supportive of this "vision". These multiple layers of teacher's practice have been downplayed in the research on teacher practice in mathematics education. This study suggests further research on these practices.

The NCTM Standards [1989] have been widely praised for the vision of mathematics teaching promoted by reformers. However, despite concrete illustrations from classrooms and articulated images and principles, they do not present a unitary set of practices. Teachers would benefit from a series of workshops that present examples of inquiry-oriented and meaningful teaching in mathematics and mathematical problem solving. Teachers who have never seen children discussing mathematics or actively engaged with a mathematical problem need to see what this looks like. Such sessions would have the potential to challenge teacher's beliefs about their own problem solving practices and encourage teachers to construct, and to de-construct, their own understandings of what they mean when they say they have an inquiry-oriented approach to mathematics and problem solving. Exploring such personal understandings could provide the necessary foundation in building such new and different pedagogies.

This study suggests professional development for teachers similar to that described by Schifter [1994]. In such workshops, teachers are challenged at their own levels of mathematical competence and presented with opportunities to experience mathematics as students. In learning this

way, teachers can increase their mathematical knowledge and experience a depth of learning that would be, for many of them, unprecedented. Such activities allow teachers to encounter mathematics, often for the first time, as an activity of construction, exploration and debate rather than as a finished body of knowledge to be accepted and reproduced.

Simply providing workshops on mathematical topics such as "real-life" connections, manipulatives and group work does not appear to have been sufficient to develop the deep understandings that are necessary in order to address such teacher's beliefs such as "doing math is doing computation".

Teachers would benefit from workshops on how they encounter mathematical problem solving in their own lives outside of school. In Language Arts' reform, teachers were expected to become readers and writers, to write in journals and to write for publication in order to understand what it is to be a reader and a writer. It was felt that such personal understandings were necessary in order for teachers to see reform at a deep level. Mathematics teachers could be encouraged to reflect on themselves as "mathematical beings" in order to understand what the reforms suggested for students' mathematics knowing.

Teachers would benefit from professional development in which they engage with parents and society on society's perceptions of mathematics and mathematical problem solving. This study has indicated that teachers do not work in isolation and the influence of parents and society is significant in affecting teacher's practice. Change in mathematics teaching and learning is more likely to occur if society has a different perspective of mathematics. Again, in reference to Language Arts reforms, teachers were encouraged to work closely with parents in order to guide them in effective mentoring

strategies for reading and writing with their children. In the process, parents often changed their view of reading and writing education. Similarly, teachers, in working on mentoring strategies with parents in mathematics, would benefit as parents' beliefs are challenged. This study has indicated that the attitude of parents is an important factor for students' attitudes towards mathematics. Considering Garofalo's [1989] research that students solve problems to get a sufficient number of problems correct in order to satisfy their parents and avoid failure, do the expectations of parents encourage a traditional "number-oriented" style of problem solving? This study also suggests a need for parents to be educated in current theories about teaching and learning in mathematics and mathematical problem solving through workshops in which they can experience the power of the NCTM "vision" for mathematics education.

Finally, teachers would benefit from some acknowledgment of and support for the profound change that they are experiencing in mathematics education. Again, in Language Arts' reforms, professional journals often present teacher testimonials about the effect that the changes in their approach to teaching practice in Language Arts has had on their professional and personal lives. Teachers change intellectually, practically and emotionally and such continuing testimonials on the part of teachers either in text or through the NCTM web site would have the potential to break down the feelings of isolation of teachers who struggle with their changing pedagogies in mathematical problem solving.

With respect to the implications presented above, future research in the area of professional development for mathematics teachers could include research on how teachers build their personal mathematical beliefs. Such research could explore why teachers like Lynne continue to present routine

problems to their students despite their "inquiry rhetoric". As for teachers' personal beliefs, research on how teachers see themselves as "mathematical beings" would include such issues as how teachers solve mathematical problems and how teachers use mathematics in their lives outside of school. In coming to understand themselves as such beings, possible understandings of deeper issues of mathematics related to problem solving could be explored.

Teachers would benefit from research on how students, parents and society build their traditional beliefs about mathematics and how school and the world outside of school reinforce such beliefs.

Research on issues of teacher accountability in mathematical problem solving could address how standardized tests, curricular expectations and expectations of parents and society effect teachers' changing practice in mathematical problem solving. In addition, other perceived constraints on change in problem solving teaching practices could be explored. In regard to issues of accountability, questions pertaining to what part of the mathematics curriculum teachers focus on and their perceived "ranking" of mathematical topics, such as computation and geometry, would address further the issue that different teaching styles and understandings accompany certain mathematical topics.

In reflecting on this study and considering how the study might have been conducted differently, I would spend more time in the classroom, looking in-depth at what the students were doing as they worked on the problems given by Lynne in her classroom. I would spend more time with Lynne, discussing her practice and exploring with her why she was doing what she was doing. This type of study would have the potential to examine more closely what actually happens in the students' problem solving groups

and in Lynne's planning and delivery of her lessons - with "natural" problems, in a "natural" setting.

It has been over a decade since the NCTM Standards [1989] were introduced to the educational community. The changes suggested by the NCTM were vast and represented a very new way of teaching mathematics. This study has suggested some direction for professional development in mathematics that has the potential to support the efforts of teachers like Lynne whose teaching practice is multi-layered and presents a continuum of practice from routine to inquiry-oriented as she endeavors to provide mathematically meaningful problem situations for her students.

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September, 1994

Dear Parents

This year, Ms. Dianne Dodsworth, a doctoral student from the University of Alberta, will be conducting research in my classroom. The research will be conducted for the entire school year.

Permission to conduct this research has been granted by both the University of Alberta, the [name of city] Board of Education and [name of Principal], the Principal of our school. All names as well as the location of the school will be anonymous.

This will be an opportunity for your child to experience learning opportunities inn mathematics. We ask your support for such research. Ms. Dodsworth and I would be more that happy to discuss this research with you during the school year.

Thank you,

....

I,...do/do not give permission for my child,......................................., to take part in the classroom research in mathematics which will be conducted from September - June , 1994 - 1995.

Signed:.. Date: ....

Appendix B Problems Presented in the Problem Solving Sessions

Sudden Wealth

Problem 1

A mysterious genie has just told you that you will receive $3 a minute for the next year.

How much will you receive?

1. in one hour __________
2. in one day __________
3. in one week __________
4. in one year (52 weeks) __________

Problem 2

With your new wealth you decide to buy a cassette tape for each student in your school. Suppose there are 746 students in the school. Each cassette costs $8.95, plus $0.64 tax.

5. How much do the cassettes cost without the tax? __________

6. How much tax will you have to pay? __________

7. What will the total cost be? __________

Problem 3

You decide to treat everyone in your class to lunch at Burger Palace. There are 29 students in your class. Big Burger costs $1.79, soft drinks cost $0.75, and sundaes cost $1.35. Each student gets one each.

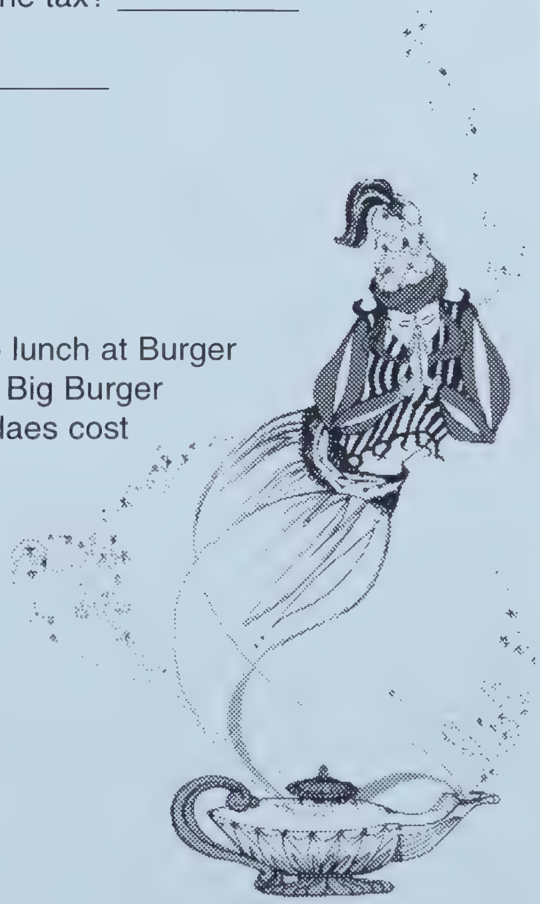
What is the cost?

8. all the Big Burgers __________

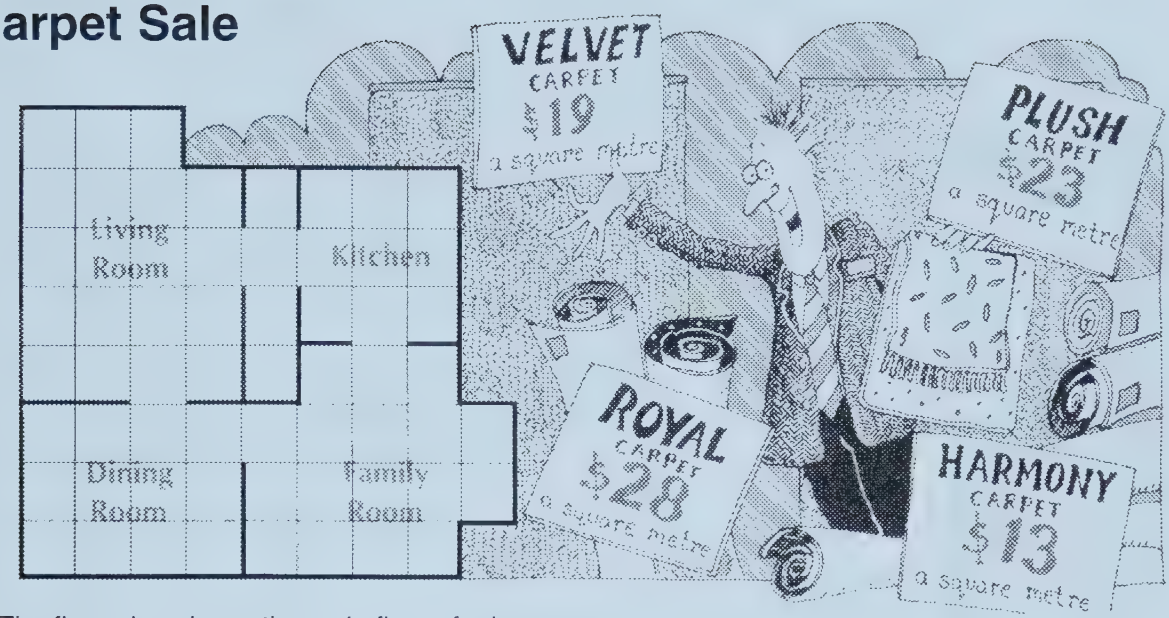
9. all the soft drinks __________

10. all the sundaes __________

11. the total __________



Carpet Sale



The floor plan shows the main floor of a house
Each small square represents one square metre.

1. Estimate the cost of carpeting the main floor using each kind of carpet

   Harmony carpet__________ Velvet carpet__________

   Plush carpet__________ Royal carpet__________

2. How much would it cost to use Velvet carpet in each room?
   Show how you got your estimate.

	Kitchen	Family Room	Dining Room	Living Room
Estimate				
Actual Cost				

At the Movies

Solve.

1. Tom had $10 when he went to the movies. His ticket cost $3.75. He bought popcorn for $1.50 and a soft drink for $0.95. How much money did he have left? __________

2. Mr. Hamelin bought 3 boxes of popcorn at $0.95 each and a package of peanuts for $1.20. How much did he spend? __________

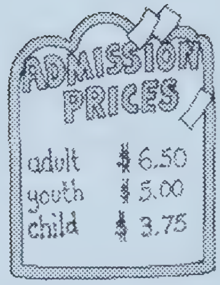
3. The 2 adults and 2 children of the Nielson family went to the movies. How much money did the tickets cost? __________

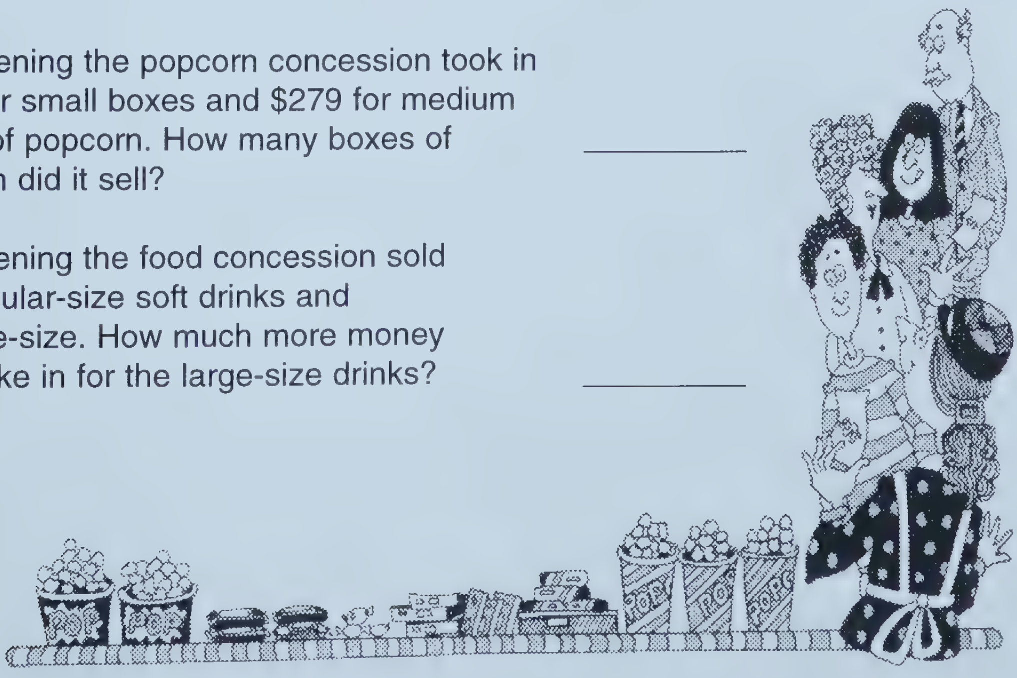
4. One evening the popcorn concession sold 275 small boxes, 183 medium boxes, and 56 large boxes of popcorn. How much money did it take in? __________

5. The popcorn machine uses 12 L of cooking oil a week. The oil costs $3 a litre. How much does the oil for 52 weeks cost? __________

6. One evening the popcorn concession took in $228 for small boxes and $279 for medium boxes of popcorn. How many boxes of popcorn did it sell? __________

7. One evening the food concession sold 126 regular-size soft drinks and 77 large-size. How much more money did it take in for the large-size drinks? __________





Appendix C Problem Solving Data

At the Movies

Problem 1

Ka	Tom had ten cents when he went
Li	Ten, ten, Tom had ten dollars when he went to the movies
Ch	Let's just all work on one here
Li	His ticket costs, wanna work all together?
Cr	Three seventy five
Ch	Ya, Ok
Li	Ok, we'll work together
Ka	Is that asposed to be taping
Ch	We're working together on one sheet
Li	Oh yeah it is, ooo
Ch	Kay let's all work on one
Li	Ok
Ch	Tom had ten dollars when he went to
Cr	To the movies
Ch	To the movies
Ch	His ticket cost three seventy five
Ka	His ticket cost three seventy five
Li	Ok
Ch	He bought popcorn for one fifty
Ka	He bought popcorn for one dollar
Ch	And a soft drink for ninety five cents
Ka	And a soft drink for ninety five cents
Ch	How much money did he have left
Cr	Ok Ok, three seventy five plus one fifty
Ka	No, no calculators

Cr Calculators

Li Sshhh

Li Ok, three fifty

Ka What's three seventy five

Li Oh three seventy five

Cr Point three five

Li Three point seventy seventy five

Ka Three point seven five

Cr Three point five times

Li Point fifty plus

Ka A buck fifty

(mumbling)

Li Times

Ka Plus ninety five cents

Li Ninety

Ch Nine five and equals, woah, ud, ud, ud a blgh, a dollar twenty five

Ka No, that's impossible

Li That can't be it

Ch Ok here

Ka You forgot to add the ten cents guys

Li It's ten dollars

Ka Sorry, to

Cr Ok, let's just do it on a piece of paper

Li Ya it's just

(laughter)

Cr Ok, so three seventy five

Ch Three seventy five

Li Kay we'll all work on it on a piece of paper

Ch Three seventy five

Cr Plus

Ka Tom had

Li Three seventy five, one fifty and then ninety five cents

Ka And ninety five cents

Li So I have to add that up and subtract it by um ten and see how much he has left

Ka And then ten dollars, Tom had ten dollars

Cr Nope

Ch No, don't you ya, you add this all together

Li Then you subtract it by ten and see how much he has left

Cr Sixteen, seventeen, eighteen, nineteen, twenty, twenty one

Li Five point five is one, thirty one, so that's eight, eighty nine, ninety four

Ka Three one four

Ch That's six, there six dollars and twenty cents

Cr (unclear)

Ch Ya, so now you have to subtract it

Li Six dollars

Ch Plus, I know

Li I think I screwed up

Ka Ok, ya, I think you did ...

Ch I got this, I got this

Ch I got six twenty after I added all this up

Ka Kay, what did you get, six twenty so it must be right

Cr (unclear)

Ka Now we have to

Li Subtract it by ten

Cr How much money did he have left

Li So we have to subtract it by ten and see how much he's got

Ka Ok, he had ten dollars

Ch Ok, so six twenty subtracted by ten?

Li Six

Ka Ya

Li Twenty by ten

Ch Oops, I screwed up

Li That's easy, four dollars and eighty cents

Ka Ya so he had four dollars and eighty cents left

Li Ya

Ch I'll write this down

Li Four dollars and eighty cents

Ka Four eighty

Problem 2

Ch Kay, I'll read the next one, no, who wants to read the next one

Ka I will, I will

Li Ok

Ka Ok Mister Hamlin bought three boxes of popcorn at ninety five cents each and a package of peanuts for a dollar twenty. How much did he spend. That's easy

Cr But how much, Ok, let's see how many

Ch He bought three boxes of popcorn so three boxes of popcorn

Ka How much did he spend

Ka Is ninety five Ok, three boxes of popcorn

Ch Of popcorn and they're each I mean each

Ka So we have to add three times ninety five cents, three times ninety five cents

Ch Three times ninety five cents

Li Three times ninety five cents

Cr I'll get it down on paper

Li Ok, [student's name], you do it

Cr Three times ninety five equals

Cr Two hundred and eighty five

Ch Two hundred and eighty five

Li Two hundred and eighty five

Ka What

Ch Two hundred and eighty five

Li Two hundred and eighty five

Ka Ok, so two hundred and eighty five

Li So that'd be two eighty five

Ka Plus uh, a dollar twenty

Li Twenty

Ch So it's two dollars

Ka Plus a dollar twenty equals

Ch Eighty five plus a dollar twenty equals

Ka Twenty eight, uh, twenty eight dollars and sixty two cents

Li So that's how much he spends

Ch Four, he spends four dollars five cents

Li I think

Ka That's not what I got

Li It can't be two hundred and eighty

(laughter)

Ka No, I said, I said make the dot there

Ch Twenty eight dollars

Ka I said twenty eight dollars and sixty two cents

Ch I only got four oh five and I added it that's way off someone's way off here

Ka Because , OK, who's

Li Oh no

Ch Three boxes of popcorn

Ka And we got that to be how much

Ch Two hundred and, two dollars and eighty five cents

Li Ok two eighty five plus

Ka Ok so two dollars eighty five cents

Ka Plus, Ok a dollar twenty

Li A dollar point twenty plus

Ka Equals four dollars

Ch That's what I got sorry

Li And five cents

Li So four

Ch Four dollars five cents read the next question

Ka Hey, I'd like to write some stuff here too

Li OK, so four point

Ch OK, I'll read the next question OK what where is. Actually, we've read two so you guys get to read one each now

Cr I've already read one

Li How much money did he spend

Ch OK, I'll read this one

Li Hey, hey, I haven't read one yet

Ch OK, you read the number three

Problem 3

Li OK, the adults, the two adults and two children of the movies, of the Nelson family went to the movies.. How much money did each ticket cost

Cr Of the

Ch Each ticket OK

Li It doesn't tell us how much, how much the tickets cost

Ch It says two adults and two children of the Nelson family went to the movies. How, how much money did the tickets cost and they don't say how much the tickets cost

Li Here it might be up here, adult and a child, right here, so we forgot about these, they're up here in the top one

Ch No

Ka Ok

Cr There's no use, OK, so

Ka Adult, uh six fifty

Li OK, six fifty

Ka Um, children, three seventy five, hold on, guys, we have to add six fifty twice

Li Oh

Ch Oh

Li OK, so it's six

Ch OK, how many children are there

Ka Plus

Ka Um there's there's there's two so plus three fifty plus three point

Ch It's three seventy five

Cr Fourteen fifty

Li Plus how much, how much was the kids

Ka Seventy five

Cr (mumbling)

Li OK, plus three seventy five, plus three seventy five

Ka OK, it's twenty twenty dollars and five cents

Cr (unclear)

Li Wha!

(laughter)

Ka I think you did it wrong ...

Cr Twelve, thirteen, fourteen, fifteen, sixteen, seventeen,

Ka I got, I got twenty dollars

Li Forgot the decimal

Ch I got twenty four dollars

Ka Kay, I'm going to do mine again

Ch I messed up, I'm doing mine again

Ka Kay

Li I don't know what I've done

Cr Twenty dollars and fifty cents

Li That's that's what I think that's what I had I don't know

Ch Let's try it again

Li OK

Ka Kay, so we have two adults, six fifty

Li OK six dot fifty plus six dot fifty plus three dot seventy five plus three dot seventy five

Ka Fifty plus six dot fifty plus uh three seventy five six, um, oh what

am I doing

Ka Hey, everyone check theirs, Ok cause we might have all an, all different answers

Cr Here, ...

Li I got thirteen bucks

Cr OK, kay, kay

Li Ya, I know what it is

Cr Six

Li Equals six dot fifty

Cr Oh I, I got it, kay, six

Ch I got twenty dollars and fifty cents

Cr Twenty dollars and fifty cents, that's what I got

Li That's that's the second one that I got

Cr Six point

Ka I got thirty three dollars

Cr Fifty

Ch I got me and ..., Ok lookit, here I'll do it again

Cr Fifty

Ch Six, six dollars and fifty cents is for two adults right

Li Cents ya

Ch And then there's three seventy five

Li Why don't you just use the times sign

Cr Three point seven five

Ka Ya, that's what we should have done

Ch And then you do five and five is ten right, put the zero, carry the one and then you add this up

(unclear)

Cr Twenty dollars and fifty cents I just couldn't add the

Ch That's like twenty five though you put a five, put a two six and twelve and then you just add eight (unclear) seventeen and I mean eighteen and then two is twenty times these (unclear) zero then you put the two

Li OK, so it's twenty

Cr Twenty dollars and fifty cents

Ka That's what I have at my original one but then I didn't think that was right so I erased it

Ch Well you were right, I get to put this one down

Ka No, I do I do

Li (unclear)

Cr (unclear)

Ka You're always writing on this

Cr OK

Ch I don't care

Problem 4

Cr One evening the popcorn concession sold two hundred seventy five small boxes one hundred eighty three medium boxes and plus fifty six large boxes. How much money did it take in.

Li OK, Large, large is two twenty five

Cr OK, so we need to do um

Li So fifty six times two twenty

Cr We need to do case

Ch What is it, fifty six times what

Li OK, they, two hundred and seventy they sold two hundred and seventy five small boxes

Ka Kay um two hundred

Li A hundred and eighty three medium and fifty six large

Ch OK so, OK so what is it, so two hundred seventy five, two hundred seventy five

L And fifty six large

Li One eighty, a hundred and eighty three medium

Ch A hundred and eighty three, a hundred and eighty three, and what was the last one

Li And then fif, fifty six large

Ch Now you have to add this all up, or times it all, add it right

Cr Twenty five

Li You have how much does the, does the oil for fifty two weeks cost

Cr So much money

Li Fifty four, fifty two weeks, times fifty two

Ka What

Li We have to add this together and times it by fifty two

Ch OK, I'm adding it

Li OK

Ka You're timesing it obviously

Li All this together by fifty two and then it says how much does it cost to um supply the popcorn at the movies

Ch OK, I've added it all and it equals ten fourteen

Li OK

Ka Ten dollars and fourteen cents

Cr Here let me get this straight again kay

Ch Ten dollars fourteen cents

Cr Seven five, is it two seventy five Wait, don't don't write anything

Li Ten dollars and fourteen cents

Ch OK, now we have to times that answer by what

Cr Twenty five

Li Fifty two

Cr Times

Ch OK, ten fourteen

Cr Zero point seven

Li (unclear)

Ch Times what Whoopsies

Li Fifty two

Ch Ten fourteen times fifty two

Cr This is the answer

Li Ya

Cr So they got two hundred

Ch That's the answer

Li (noise)

Ch That's the answer

(laughter)

Cr Fifty two thousand seven hundred twenty eight

Li Dollars to supply popcorn, fifty two weeks, I'd think about it

Ch That sort of, the answer is

Li Think about it, fifty two weeks

Ch Fifty two, is that all that it says

Li Ya, oil

Ch For fifty two weeks

Ka Um for fifty two weeks, I don't see it here

Cr Wha, what question are we doing kay, what question

Ch We're not on that one, we're on this one

Ka On this one

Li Oh

Cr Oh

Li Oops

Ka One evening the popcorn session sold two hundred and seventy five small boxes, one hundred and eighty three medium boxes and fifty six large boxes of popcorn. How much money did it take in

Ch I've added that all, I've added that all

Ch And I added all those numbers together and I got ten fourteen

Ka How much, how much money

Ch That means how much did they get

Li How much did they get out of selling it all

Ch Ya, and I got ten fourteen

Ka I don't know

Li Oh

(laughter)

Ka OK

Ch They've added two seventy five

Ka OK let's see

Cr Should we go to the next question and come back to this

Ch No

Li Let's just write down the answer and we'll come back to it

Ch Ten fourteen, ya, I think, ya cause that could have done, been it, ten dollars fourteen cents

Li Ya, OK

Cr I doubt it

Ch Well, I added two dollars and seventy five cents

Ka It, OK, I got five hundred and hold on guys I got five hundred and four

Cr That is not the answer

Ka And I just did it on

Ch That's five dollars and fourteen cents

Li It's not two dollars and seventy five cents, it's two hundred and seventy five small boxes

Ka Oops, sorry

Cr Kay, so it should be two hundred seventy five groups of ninety five equals

Li A hundred and eighty three medium boxes, fifty six large boxes of popcorn, how much money did they take in it's a lot more than ten fourteen

(laughter)

Ch Twenty six, no, two hundred and sixty one dollars and twenty five cents

Cr No, it didn't have a point

Li No

Li Twenty six hundred one hundred and

Cr This is how much cents

Li No it doesn't make sense

Cr That's how much cents

Ch I got ten fourteen

Li Twenty six thousand, one hundred and twenty five

Ch Let's go back to that one after, put a circle around thatbox so we know to go back to that one

Li Twenty six thou, twenty, twenty six thousand, one hundred and t twenty five

Ch Let's let's go on to a different one

Li OK, we'll just circle this

Ka Cause that was taking too long

Problem 5

Ch OK, how bout this one, the pop, the popcorn mach, the popcorn machine

Ka One evening the popcorn (unclear)

Ch The

L Number five

Ch The popcorn machine uses twelve liters of cooking oil a week

Li Twelve liters of cooking oil

Ch The oil costs three dollars a liter

Cr Dollars

Ch How much does the oil for fifty two weeks cost

Li (noise)

Li I think you have to times twelve times three

Ka I got sixty seven I don't know what that's supposed to mean though, sixty seven

Li Plus fifty two

Cr OK, Twelve times three plus fifty two equals eighty eight

Ka But I already did that, Ok you're timesing it

Li Twelve times three plus fifty two, eighty eight dollars

Ka Twelve times three equals

Ch That's the answer Kay, let's put down eighty eight dollars

Ka That was sort of an easy one

Li OK, Oops, wrong thing

Ch Kay, how's eighty eight zero zero, hey, they got a lot of money

Li Oh ya

(laughter)

Ka I have more than that

Problem 6

Cr OK, one evening the popcorn concession took two hundred twenty eight for small boxes and two hundred seventy nine medium boxes of popcorn. How many boxes

Ka I have way more than that

Ch How much do you have

Li Two

Ka I have more like three hundred more than that (unclear) ninety eight

Ch I have three hundred ninety

Li How much was it

Cr OK, twenty eight two hundred twenty eight

Li Two twenty eight

Li Two twenty eight

Ka Two

Cr Divided by um

Ka Divided by

Cr Dvided by ninety five

Ch What are we on

Li Nine five equals

Ka Nine five

Ch What are we on

Ka Two dollars and four cents?

Li Two dollars and four cents

Ka Hunh

Cr No, kay, kay

Ch Kay, kay, kay

Ka Something isn't quite right

Cr Kay, two hundred, two

Ch What is it

Li Two eight, two twenty eight

Ka Two hu

Cr Twenty eight divided by

Ka Two hundred and seventy nine, that's what we got mixed up on

(laughter)

Li Oh [student's name]

Ch [Student's name] did it wrong

Ka Divided by

Cr I did it

(unclear)

Ka Two seventy nine divided by

Ch She got it wrong

Li Well, of course

Ka Two argh, seven, seven, seven

(laughter)

Ka Kay

(laughter)

Ka Two hundred twenty eight, or twenty nine, twenty eight right

Ch Un hun

Ka Kay, divided by

Ch Two seventy nine

Ka Two seventy nine

Li Two seventy nine

Ka Two seven nine

Cr Divided by

Ka Equals

Li Equals

(laughter)

Ka Ooooh

Li Ooooh

Cr I got zero eight one seven two oh four three

Ka I have different than that

Li I won, I won, I won a million bucks, that's the lotto number today (laughter)

Ka I didn't get that, I got , I got zero zero, zero point zero zero three five eight four two

Cr Zero zero, how dare you have a wrong answer

Ka How dare you, OK, let's do, la we'll do it again

Cr Kay, on

Ka Two twenty eight divided by uh two seventy nine, two seventy nine equals

Ch Ha, let's do this on paper

Cr Two twenty eight divided by two seventy nine

Li Two twenty eight divided by two seventy nine

Ka I was right

Cr Zero eight one seven two oh

Ch But this is money aren't we

Ka Four three

Cr Four three

Ka How are you supposed to

Li I don't know

Ch Well, isn't this money, isn't this money

(unclear)

Ka Ya

Li Ya, it's money

Ch Ya I know

(unclear)

Ka This is so confusing

Li Oh yeah

Ch Let's try doing it on paper

Ka Ya

Cr I really I think (unclear)

Ch 'm adding I might add, I might, I'm going to try adding it

Li Ya that's that's might work, maybe that's what we're doing wrong with the calculators or something, I don't know what I'm doing so

Ka Kay, well let's try that then, two two two eight plus

Ch I got five dollars and seventy

Ka Um three two seventy nine equals

Li Two seventy nine equals

Ch I got fifty dollars and seven cents, I added it and I got

Ka I got five hundred and seven dollars

Li Seven nine plus

Ch I got fifty dollars and seven cents

Ka Kay we'll all do, do that because it's got like the same thing so I guess

Cr Fifty two

Li I don't know I got five hundred and three

Ch I got, I did it plus and I got five hundred seven

Ka I got five hundred and seven and she got five hundred and seven

Cr What do you do, what do you do

Li Then it's probably

Ch I got five hundred and seven

Ka So it's five hundred seven

Cr Two twenty eight plus what

Ch I got five hundred and seven

Cr Two seventy nine

(unclear)

Ch I added it, five oh seven, it's five oh seven

Li Five oh seven

Ch Or

Ka See, we were right, we were always right

Cr No, but that's only for this one

Ka I know

Cr Kay now we need to do

Ch You write it down, you write it down, write the answer

Ka Oh

Ch Kay, what's the answer

Cr You were supposed to, OK, OK, OK

Li Five hundred and seven ya

Ka Ya

Cr Ya

Ch Ya

Problem 7

Ch Kay, we're doing this one, we're doing number seven we're doing number seven

Li One evening the food concession sold one hundred and twenty six regular soft drinks and seventy two large. How much hey, hey, how much money did it take for alarge size drink.
(laughter)

Ka OK, OK, here are the softdrink prices, kay, a large is a hundred
(laughter)

Li A dollar

Ka A dollar fifty so on a dollar fifty

Li Twenty thousand four hundred and fifty five equals

Ch Kay (unclear)

Ka Plus two two

Ch Tutu (laughter)

Ka I mean two two

Cr Hey, did we finish this one

Li I don't know

Ch Yes we did and we got the answer was two oh seven

Ka Ch hundred and twenty six, a hundred and twenty six

Li Two oh seven

Ka (unclear)

Li It was five oh seven

Ch It was five oh seven, you got that all wrong you put two oh seven

	and it's five oh seven
Li	It's five oh seven
Cr	Fifty dollars and seven cents
Li	Five oh seven
Ch	We were wrong, fifty dollars seven cents
Ka	Plus (unclear)
Ka	I got it, two hundred and four dollars, that's definitely wrong
L	Uh ya
Ch	One evening the food concession sold one hundred and twenty six maybe you added it
Ka	I did, that's how I just did it
Ch	OK, OK, try timesing it
Ka	OK, what is it
Ch	There's only four ways you can do it right
Ka	Two hundred twenty six
Ch	No, no, no, no one hundred twenty six
Ka	Oh
Ch	Maybe your adding was wrong did you put one hundred twenty six
Ka	No, I put two hundred twenty six
Li	(unclear)
Ch	And it's only one hundred twenty six you'll only be a hundred off
Ka	OK, so one hundred twenty six
Li	(unclear)
Ka	Plus
Li	How much money did it take for the large size soft drink
Ka	Seventy seven plus a dollar fifty, a dollar fifty, I still have two hundred and four dollars

Cr Seven nine ten, that's ten

Li Maybe that's the answer

Cr No, one

Ch I got twenty dollars and three cents

Li That's how much more

Ka That's not what I have

Li That's how much more the large size costs

Ch I got twenty dollars and three cents

Li Than the, than the regular

Ka Alright, have you been adding that

Cr Kay, what did you do, what did you do

Ka Cause that's what it at, that's what the large is like you're that's what the problem is you forgot to add a hundred and fifty

Ch OK, so what is it, one hundred twenty six, one hundred twenty six um

Ka One hundred twenty six

Li One hundred twenty six

Ka Plus

Li Plus

Ka A dollar fifty, a dollar fifty plus

Ch Plus hmmm

Li A dollar fifty

Ka Seventy seven equals, I still have two hundred and four oh five

Li Seventy seven

Ch It doesn't work

Li OK seventy seven

Cr OK wait, let's get that (unclear)

Li Kay, one three five eight plus seven

Cr I got it, I got it, it's two hundred dollars

Li (unclear)

Ch Nope thirty seven and three, thirty seven dollars three cents

L Fifteen

Ka We have all different answers

Ch Cause you have to put this down cause this is how much they cost

Ka Ya, a dollar fifty

Li I got three hundred and fifty three

Ka Hold on guys

Ch I got thirty

Li I got three hundred and fifty three

Ka OK, it says one evening the pop, sorry

Ch (laughter)

Ka Um, in one evening the food concession sold one hundred and twenty six lar, regular size

Ch And then I put regular

Ka So you what you're doing is you're timesing it, one hundred and twenty six to one hundred and fifty

Ch OK, we'll do that

Ch One hundred and twenty what was it One hundred twenty what, six

Ka One hundred twenty six times one hundred and fifty

Li One twenty six times one fifty

Cr How do you get that

Ch I got this number

Cr Six times

Ka Then

Li Then what

Ka Then seventy Ok, then it says regular size soft drinks and and then it says seventy seven large size

Ch So now we times this again

Ka So times seventy seven times a dollar fifty

Li Seventy seven a dollar fifty

Ch I lost that other answer

Li Seventy seven times a dollar fifty

Ka Oh well she's still got it

Cr I got...one thousand, one hundred sixty

(laughter)

Cr Let's see ...

Ch I got this

Cr Well let's see

Ka That looks more reasonable

Cr One thousand one hundred

Ch One hundred

Li That's how much more the large was

Ka Kay, you guys, do it again cause you guys

Cr That's the same one I got

Ch Do it again, I don't think I want to

Cr Almost the same, almost the same

Ch [laughter)

Ka Do the, do the calculations again so

Ch OK give it to me again

Ka OK, it is twenty, two hundred, one hundred and twenty six times one hundred and fifty, I mean a dollar fifty

Li Times a dollar fifty

Ch A dollar fifty equals, write this number down

Ka Plus seventy seven

Ch Eight nine plus

Li OK, plus seventy seven

Ka I mean, I mean times seventy seven

Ch times, times

(laughter)

Ka Equals

Ch I think we got a wrong number here

(laughter)

Ka That's what she had last time

Li I got a hundred and ya that's what I had, you do it like me

Ka Kay, do it again guys

Ch I don't want to

Li I don't want to

Ka OK, I'll do it

Li OK [student's name]

Cr OK, what is it, you want to do it

Ch Who you going to pick

Ka OK, OK

Cr What is it

Ka One hundred twenty six, one twenty six times a dollar fifty, um kay now seventy seven

Li (unclear)

Cr Plus seventy seven

Li I don't know

Cr Plus seventy seven plus seventy seven

(noise)

Cr I got two sixty four

Li Write it down

Cr Look at this, I like doing this

Li I like doing this

Cr One evening the food concession sold

Ka One evening the food concession sold one hundred and six regular size soft drinks and seventy seven large size

Ch Kay, the last one is this is the one we were having trouble with

Ka How much more money did it take um in for the large sizes

Ch How much more, we were thinking

Li OK, so it's one twenty six uh times how much are the medium

Cr Seventy seven

Li Seventy seven, no

Ch Zero point seven seven

Li Times a dollar fifty

Ch It's on the memory side

Li One fifty equals, kay altogether the smalls cost a hundred eighty nine dollars

Ka I got nine hundred, seven thousand, I mean, nine ten thousand seven thousand, a hundred, zero

Cr Kay [student's name]

Li I got a hundred eighty nine for the medium soft drinks

Ka I got I can't I don't even know how much that, I think I did mine wrong

Ch OK

Cr OK, and how much did you do for this

Li I didn't do that one yet

Cr Kay, do the seventy seven one

Li OK

Ch I spelled hello, H E L L O

Li OK seventy, seventy seven times

Ch H E L L O

Li How much are the large

Ch I did, I spelled hello

Li Large

Ch Hey, camera you wanna see how you spell hello

Li Large

(laughter)

Ch Wanna see how you do it, I mean

Ka No guys, guys

Li Oh crap, I missed that

(laughter)

Ch Nice, I hope the whole class hears that

(laughter)

Ch [Student's name]

Sudden Wealth

Problem 1

#1

Cr Sudden wealth

Li Sudden wealth

Ch Oh do we get money

Ka I'm wealthy now

Ch Oh, Alladin's lamp

Ka OK

(yelling)

Cr Guess what [researcher's name], we started a year end show, we're doing a dance for Alladin

Ka A mysterious genie

(singing)

Ka Uh, is it plugged in? Oh it's in , it's on, it's on

Ka A mysterious genie has just told you that you will receive three dollars a minute for the next year. How much will you receive in one hour

Li Three times, three times sixty

Ch That's easy a hundred and twenty

Ka A hundred and eighty

Ch A hundred and eighty

Li A hundred and eighty

Ch I write the answer down

Cr A hundred eighty

Ch No, whoever figures it out gets to write it down

Ka I did

Ch So you get to write down the answer

#2

Cr OK, in one day

Ka In one week

Cr Oh that's in one hour

Ch In one day

Ka Sorry in one day, in one day three, three dollars

Cr Three times

Ka Oh no

Cr Three times um

Ka hree times twenty four

Cr No

Li Three

Ka Seventy two

Ch Sixty five

Cr Seventy two, ha ha

Li Seventy two

(unclear)

Ka I figured out the answer

Cr Well sorry

Ka Putting it in the wrong spot

Li Wait a minute you guys, there's something we're doing wrong here

Ka How do you know Mrs. Smarty

Li Just let me see your sheet, look we're doing how three dollars each minute in one hour you wouldn't make a dollar eighty because he um because in an that'd be like every hour or something we're doing it in threes

Cr I know we did it wrong we went three point zero zero

Li So it'd be three so it couldn't be

Ch It couldn't be one eighty

Cr Times

Ch It can't be

Cr Times

Li Because you get three dollars a minute

Ka So three, three times, three, kay one hour

Ch Three times sixty

Ka Well, there's sixty minutes in an hour so three times sixty

Li OK

Ka A hundred and eighty

Cr Ya, I know a hundred and eighty

Ch Hundred and eighty

Ka That's right

Ch Look at this, look at this

Li Well it can't be a hundred and eight cents

Ch A hundred and eighty dollars, hey that's rich

Ka Ya, I know sudden wealth

Cr Kay in

Li In one day

#3

Ch OK in one week so that's um

Li Kay

Cr Kay

Ka We've already done that, you guys are behind it's seventy two dollars

Cr That's less, that's less than in hours

Li That's less than just an hour we need to times this a hundred and eighty times twenty four

Ka No hold on sixty

Li One eighty times

Ka I'm going to do something

Li Twenty four

Ka Sixty times twenty four equals

Ch	Five oh four
Cr	Four three two one
Ka	Well, I'm doing something
Li	Four three two oh that's how much you make a day because
Ka	No four three two oh
Li	(unclear) one hour
Cr	Ya
Ka	Oh
Li	One hour makes a hundred and eighty so twenty four you just times it
Ka	I, I, I well first I put like um sixty times
Ch	You guys work on that one, I'm
Ka	First I put sixty times twenty four
Ch	How much money
Ka	Then I put equals, then I put times three and I got the answer
Ch	How, oK, let's work on one week
Li	Kay, four three two oh times seven
Ch	What do you get on one week
Ch	One week, one week, one week
Li	Times seven equals woah, just times it by times four three two oh by seven
Ch	Four three two oh
Ka	Four three
Li	Oh times seven
Ka	Four three two oh times
Ch	Three zero two four zero
Li	Yup, that's what everyone got

Ka Hold on, but that's for a week guys

Ch You didn't get the same as me

Ka No, kay what was it

Ch Four

Ka Ya

Ch Three two oh

Li Three two oh times seven

Li Seven days in a week, each day you get four thousand three hundred and twenty dollars

Ka Ya, I think that sounds right

Cr Kay three zero zero, oh

Ka That's about about as much as my dad gets

Li How you just times this by a week

Ch Zero

#4

Li Kay fifty two, times the last one by fifty two

Ch You do it

Li Times fifty two

Cr I mean, I got it I got it

Ka What, what, times it by what, times it by what

Cr I got it anyway

Li Fifty two times

Ka Times it by fifty two

Li Three thou what a thirty

Ka That's that's wrong

Cr No it isn't

Ka Oh you mean for in one year

Li Thirty thousand two hundred and forty times fifty two cause it

Ch That's right

Ka Ya I thought we were still on one

Ch One five seven, one five seven

Li So it's it's this

Ch Two four eight zero

Ka Two four eight zero

Ch Zero

(laughter)

Ka No, that's right

Cr And kay we finished that question

Ka OK

Problem 2

#5

Ch OK I read the next one cause you read that one, with your new wealth, you decide to buy

Ka A cassette

Ch To buy a cassette tape for each student in you school suppose there are seven hundred and ei, seven hundred and forty six students in your school

Ka Forty

Ch Each cassette costs eight dollars and ninety five plus sixty four tax

Ka Eight nine five

Ch No, no, no but you haven't got the kids yet

Li Eight point ninety five

Ka Woah that's wrong

Cr Eight

Li Times point sixty four equals so it that I just timesed that

Ka Eight times

Ka I got five dollars and twelve cents

(laughter)

Li Times that , ok you just add the, you just add these two together and then just times it by how many students there are

Ch I got it I got it I got it

Cr That's how much

Li No it's not it's this

Ch I got the answer I got the answer

Ka How do you know it's the answer

Li Times it by seven four six

Ch Because (unclear) the numbers at the bottom

Ka Your new wealth

Li Seven four six

Ch Does it look like a genie, it looks like a guy with fish eyeballs

Li Do you have to, you have to add these um, two you have to add these two amounts together and times it by this number

Ch OK, what is it

Li Add these two together then times by the number

Ch OK, kK so what is it eight ninety five,

Li Times sixty four cents

Ch Eight ninety, eight ninety five times what

Li Sixty four cents

Cr No it's plus

Ch Sixty four cents

Cr You need to find out how much that is altogether

Li Oh ya, plus

Ch OK so what is it, what is it I can't see

Li OK

Ch Eight ninety five, eight ninety five

Cr That's how much it is altogether

Ka How much did you get

Ch Plus fifty

Ka I got , I got all this

Li How much do the cassettes um cost without tax

Ch Zero

Li It's eight ninety five, eight point ninety five

Ch Fifty four

Cr Times seven forty six

Li Times seven forty six

Ch Seven

Ka Seven forty six

Ch I've got the answer

Cr Nine times

Ch I got the answer cause you guys didn't put the decimals to say

Ka Hold on guys, just wait, just wait

Ch To say

Li Eight point

Cr That's how much it is, I put the decimal

Li Nine five

Ch I put the decimal

Li Nine five

Ch I'm writing down the answer, you have to do the decimals

Li Times seven four six equals

Ka I that's what I always do

Ch Eight seven five

Li (unclear) the same answer for every question I do

Ch I got this and me and her got this and we used decimals

Ka What did you get

Ch I got what . [student's name] got to

Ka OK, show me, show me how you do it

Ch OK, what you do is go eight dollars

Ka No, show me on the calculator

Ch OK eight dollars

Ka Kay

Ch And how much is it, ninety five cents

Li How much tax do you have to pay

Ka Plus ninety five cents

#6

Ch Plus, no plus zero point sixty four

Li Kay, point sixty four

Ka Zero point sixty four

Li Times seven four six

Ch You've got all mixed up

Ka OK let me try it again

Cr OK, start over

Ka OK what is it

Li It'll cost four hundred and seventy seven dollars and forty four c cents for just the tax

Ka Eight

Ch Four eighty five

Ka Eight four

Ch No, eight no eight OK clear, eight

Ka Eight

Ch Got ninety

Ka Ninety five right

Ch Ninety five

Li How much tax you have to pay for the whole school so that isn't right [student's name]

Ch Plus zero point sixty four

Li This isn't, how much tax, we have to pay for the whole school

Ch Sixty four times seven four six equals

Li So just sixty sixty four times seven four six

Ka Ya

Cr Sixty four

Ch Got it

Cr Times

Ch We got it me and her and you and me and you and

Li See, OK

Cr That's how much tax

Cr Tell me

Li Then there's tax

Cr Four seven

Ch That's not the answers

Li Seven

Ch They're writing the wrong answer

Li How much tax will you have to pay for the whole school not just the

money just the tax

#7

Ch Oh you're on the next question?

Li Ya

Ch We're not there yet

Li Four seven seven four

Ka Guys you guys are going ahead of us

Cr We are

Ka Ya

Li OK

Ch We're what question are you on

Li How much was the total cost (unclear)

Ch You're three questions ahead of us

Ka Without the tax

Ch How much does it cost without the tax what'd you get, is that

Ka Divide OK, take away

Ch Eight ninety five how much does the (unclear) cost without the tax

Ch Eight ninety five

Ka Can't be for the whole school so it's eight ninety

Ch That's not that's not eight ninety five

Ka I got seven oh nine oh fourteen

Li Eight point

Ka Because you have to take away the tax

Li Ninety five

Cr Ninety five

Ch Ya that's right

Li Times seven four six

Ch	Seven oh nine we got the answer we got this, OK let me try how do you, OK so what is it
Ka	Well you do what you did here
Cr	Tell me the number
Ch	OK
Li	Six six
Ch	So eight ninety five
Cr	Seven
Li	Uh six seven, six point seven
Ch	Um plus or times
Ka	Uh I think it
Ch	Times seven forty six
Ka	Ya
Ch	Seven forty six equals six, six
Ka	And then you take away, hold on, then you take away that's not right
L	OK now
Ch	That's what we got
Ka	You forgot to include the GST include the GST
Li	Eight point ninety five plus point sixty four equals oops
Ch	No we're on this question
Ka	I know
Ka	Owch the tax
Ch	Owch the tax
Ch	I did it without the tax
Li	Eight
Ka	That's what I did too and I got a different answer
Li	Point ninety five

Ch Well try yours again

Ch Might be a good idea

Li Plus point six equals

Ch One try is worth a thousand explanations

Li Plus point sixty four

Ka You guys you guys are way ahead of us I think

Cr But we're just trying to

Ka Ya guys it's not fair to us though cause you guys are gonna figure out all the answers

Li Times

Ch That's OK

Li OK this is the last one

Ch You can just copy the last one

Ch You guys are on the last question

Li (unclear)

Ch Oh

Li Just write it down

Ch We don't get it, I'm in a conference room

Ka I'm not

Ch You don't know

Li We've done this about five times while we're waiting for you

Ka Times plus

(8 seconds silence)

Ch That's it We got it I got it how much tax will you have to pay, what'd you guys get, seven one five

Li Seven four seven

Ka Guys this is so confusing every one of us kay

Cr Got a different answer

Ka We've all got different answers and you guys are rushing ahead of us and it's not fair if you guys figure out all the answers and we just copy you

Li You guys need an (unclear) then

Ka What

Ch Yes well you're way ahead of us so you're beginning have to wait for like twenty five thousand minutes
(laughter)

Ch OK now on this one four seven seven four four

Ka Four four

Ch You decide to treat everyone

Ka No, no, look

Li Wait the camera's

Ka It's [student's name] turn

Cr But I'm I'm writing it down so you guys can go

Li Me me me

Ch OK

Ka Nope nope it's [student's name]

Li I haven't had a turn

Ka No it's my turn

Li I haven't had a turn

Ka I know

Cr Ya (unclear)

Ka You decide to treat everyone in your class to lunch at Burger Place, Palace there are twenty nine students in your class. Big burgers cost one seventy nine, soft drinks cost sorry seventy five cents and

sundaes cost a dollar thirty five. Each student gets one of the one of each, oh that's a cinch, you just plus all of them

Ch I got the answer

Ka I don't quite think so

(laughter)

Ka Twenty nine

Li Oh right

Cr Twenty nine

Ka Plus a dollar seventy nine

Li Twenty nine, plus

Ch That's going in the computer I hope you know

Ka I know

Cr Plus a dollar twenty

Li It's thirty dollars and seventy nine cents, nope

Cr Times

Ch Hello, anybody in there

Li Nope, my brain's empty

Ch Have you been on TV They did something (unclear) they went hello anybody in there and you said ya somebody's been to (unclear)

Ka I got, woah, I got one hundred and seventy dollars

Li Woooo

Ch That's insane, let me try, OK

Ka You just plus all of them, you plus all of them? but I plussed all of them

Ch Twenty nine students plus one seventy nine

Ka Plus plus what

(silence 14 seconds)

Li Ya

Ka See ah now I got twenty five dollars and eighty nine

Ch I got thirty two dollars and ninety eight cents

Ka OK, thirty two ninety eight

Ch Did you use decimals, did you use decimals

Ka OK, no guys, guys, guys, guys guys, guys, guys, guys, guys, guys, yes, yes, yes, yes, yes

Cr You guys let me

Ka Let's do it again, let's do it all over again

Ch I don't want to

Ka Twenty nine, shut up

(laughter)

Li A dollar seventy nine

Ch What an insult

(laughter)

Ka Plus a dollar seventy nine

Ch Calling people in this country

Li No, a dollar thirty five

Ka What

Ka Seventy five cents

Li Thirty times seven point

Ch Is that the answer

Cr No

Li Seventy five

Cr Three point (unclear) times twenty

Li Ugh

Ka Ya, oh I got your answer do your do your calculations again

Ch OK

Cr Guess what, I got it

Ka I got it

Cr What

Ch This

Ka That's wrong

Li Eleven thousand three hundred and fifty eight

Cr You tell me the numbers, you times the, this is each meal this is (unclear)

Ka No

Cr Lookit, plus, this is how much it cost for each meal, and then you times that by twenty nine because there's twenty nine kids in the class

Li OK three times twenty nine

Ka One

Ch I got, I got, you're four off of my answer you're four off of my answer, aw you're one off of my, you're like four off

Ka I was using it first I, I had to, I had to bring it closer

Ch I'll tell you the numbers

Ka Dollar thirty, dollar thirty five

Ch Seven um seventy nine where are you, thirty five and then zero seventy five

Ka Zero seven five and twenty nine

Ch And twenty nine

Ka See, we,[student's name] and me were plussing it all together and nine

Ch No, five ten and ten and ten

Li (unclear)

Ch Two thousand and ten

Ka Eighteen, ten and eighteen ten and eighteen is eight twenty eight so

Ch Twenty eight

Li So it's a dollar seventy nine, a dollar point seventy nine times kay it's fifty one ninety one

Ka Two eight (unclear), kay seven seven and seven three four five five and seven and

Ch Seven fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty one

Ka All together

Ch Yes, thirteen fourteen

Ka Twenty three, twenty four, twenty five, twenty six

Li All the drinks

Ka Twenty six plus

Ka Now we have a four dollars and sixty eight cents

Li (mumbling)

Ch I got thirty two dollars and let me try, let me try, kay give me the numbers again what were the numbers

Ka Kay, the numbers

Ch Are

Ka Are

Ch One dollar

Ka No not one dollar, a dollar seventy nine

Ch Hey we're missing something

Li OK

Ch Remember what we did up here remember what we did up here remember, we added the money together first and then we added the

money together and then we timesed all the students, three sixty seven

Li OK a dollar kay thirty five times two nine equals ok

Ka Hey you got my

Ch No but, but it's my idea

Ka I know I'm not doing it

Ch I know you're not

Li OK we finished that one

Ka No

Ch Seventy five plus

Ka Because you got [student's name]'s got it she say we plus everything except for the children but we times the children

Li This is what we got

Li Um that's what we did for, for this the big burgers are fifty one, the soft

Ch Times

Ka [student's name]

Ch I got seventy three dollars and sixty six cents

Ka That sounds right to me

Cr For what for what

Ka Tha that sounds that sounds that sounds more better than that, fifty one

Cr What remember what we did up her remember what we did up here we added the money together and then we timesed all the children

Li That's what we did

Ch We added up the money, what'd you get

Ch We got I got this one

Cr For what the burgers

Ch Yup

Cr Kay let's times

Li OK a dollar seventy nine

Cr Kay one point seventy nine

Ch Plus

Cr Times

Ka No

Ch No it's plus

L Just for the burgers

Cr Times twenty

Ka That is not right, that is not right the total

Li It's fifty one ninety one

Ka That's not right for twenty nine kids

Cr Yes, it is cause you need to buy a sundae, a soft drink, a um

Ka A hundred

Li Did you just add these all together

Cr A burger

Ka But a hundred

Ka That's a hundred and one dollars

Cr Including the teacher though too

Ka Ya I know but how'd you know the teacher came with them and how'd you know that there wasn't two or three or four teachers

Cr Well, it says

Ch You guys children why are you adding a teacher

Ka Ya I know, ya

Ch A teacher isn't a children

Cr I didn't a add a teacher I just I just put like

Ch I let's work together and they work together you get all the answers I don't care

Carpet Sale

Problem 1

Li hello

Ch hello

Ka hello

Cr hi

Ka OK, do we turn our sheet over

Li This is[student's name] a neat person

Ka OK

Li Thank you

Ka I get to read first

Li Fine

Ka The floor plan shows the main floor of a house each small square represents one square meter

Li Ooh small house yup

Ka (unclear) twenty number 1

Ch Estimate the cost of the carpeting the main floor using each kind of carpet

Li The harmony carpet's thirteen

Cr Kay what are we su

Li Harmony carpet thirteen dollars

Li Estimate cost of carpeting using

Ka Thirteen

Cr Every, all the way through the house

Ka	Well., guys, guys, guys don't always, I knew this was going to happen again, don't, you have to explain stuff to us too
Cr	OK
Li	OK, so the harmony carpet is thirteen bucks right
Ka	Harmony carpet, where does it say that? OK
Li	Right by the stupid guide
Ka	Harmony carpet is thirteen dollars, plus
Ka	Kay the plush carpet
Ch	Plush carpet is twenty three
Ka	Twenty three
Li	Thirteen
Ch	Plus
Ka	Plus
Ka	The velvet carpet is nineteen
Li	Plus (unclear)
Ka	Plus the royal carpet which is twenty eight
Li	Plus (unclear)
Ka	Eight, eighty three dollars
Cr	No, but that's only
Li	Nineteen
Cr	Ask what
Li	Eighty three
Ch	Eighty three
Li	Get rid of your hello
Cr	Ya. you guys, you guys,
Ch	Eighty three dollars
Cr	Scuse me

Li What

Cr See um what you need to do is you need to count all these

Li OK

Cr And then, and then the um, multiply them by how many squares there are

Li OK, so we've got the answer to this, now we can just add, count these up and add it to this number OK

Cr No, cause we have to count the squares (unclear)

Li Count up, count them and then we can just plus it by eighty three

Ka Kay, in the kitchen there's nine

Li OK

Cr Kay

Ch Kitchen there's nine

Li Kitchen there's nine

Ch So

(silence 20 seconds)

Ka Fifty five

Ch Yeah, fifty five

Ka Four thousand, five hundred and sixty uh, five

Ch Sixty five

Li We got sixty one you guys

Ch We got one three, I got one three eight

Li Times sixty five

Ch I got one three eight

Ka OK

Li Times sixty

Ka Kay, let's count em again, one, two,

Cr ust a minute

Ka hree, four, five, six, seven, eight, nine, ten, eleven

Ch Two, three, four, five, six, seven, eight

Li Equals fifty one

Cr No

Li Plus sixty one. I got one hundred and forty four dollars
(unclear)
(mumbling 13 seconds)

Ka Ya, I got sixty one now too

Cr Kay this is how much it would cost to carpet the whole house with Harmony

Ka Hold on, just wait sixty one

Li Oh, OK thirteen sixty one

Ka Plus nineteen dollars

Ch Plus nineteen dollars

Ka Plus ninety three dollars plus thirteen dollars plus twenty eight dollars, a hundred and forty four

Li Uh

Cr This is what I wrote kay

Ch One hundred and forty four dollars

Cr I counted these up, so if you um if you did the whole house with Harmony carpet

Ka I'm going to do it once more

Cr It'll be seven hundred ninety three dollars

Li Kay, so we know it's sixty one now we just have to times it with all these kay

Cr Sixty one

Li And the price, the price with sixty one

Cr I timesed it with that

Ka I got a hu, I still got a hundred and forty four

Cr No but

Ch Same

Cr No, but here, let me, let me try this here

Li Nineteen

Cr What you need to do is you need to go

Li Nineteen plus

Cr Thirteen Sixty one

Cr hree point zero zero, eighty times

Ka How'd you get that

Cr Sixty one (unclear)

Ka Oh, cause it's dollars

Li uh

Ch I know

Ka But you're only, [student;'s name] you're only (unclear)

Li (unclear) Velvet (unclear)

Ka You're only um doing this one

Ch Ya, ya you're only doing the Harmony carpet

Ka You've got to do all four

Ch You have to do all four

Ka And it, it costs a lot more than seven hundred and ninety three bucks

Ch Ya

Ch Because twenty eight plus nineteen plus thirteen plus thirteen would be (unclear)

Cr That's how much it would be, that's how much it would be

Ka Well, I'm gonna, I'm gonna try it

Cr You guys this is how much it would be to carpet the whole house with um Harmony

Ch Nineteen dollars

Li Now nineteen six, nineteen times sixty one, holy

Cr It's estimate, you're not supposed to find it out

Ch I know that

Li It's estimate down here

Ch Now let's see um aw, I forgot Harmony carpet,

Ch It's the Harmony carpet Is

Ch IT's, um, the Harmony carpet is thirteen dollars Thirteen sixty

Li The velvet is eighty

Ch The vel, ya, the velvet, no, nineteen

Li Eighty, that's how much it cost to cover the whole house it's eighty

Ch Oh

Cr Not quite, I don't think so

Li I just did it on the calculator

Ka Eighty

L [Student's name]

Cr Well, what did you do

Li Watch closely, one nine plus eighty dollars six one equals

Cr But that's plus not times

Ch And if you do the whole house with harmony carpet it'll probably be ninety three

Li Uh

Cr That's only one square

Ch Would it be ninety three

Li So then it'd be one thousand one hundred and fifty four cause I tried that .[student's name]

Ch Nineteen point zero zero

Cr That's how much it would be about

Ch Wouldn't the harmony carpet if you did the whole house with Harmony carpet be ninety three dollars

Li No

Cr No

Li Seven ninety three

Cr Seven ninety three

Ka Kay, well, this I, I personally, I personally think that this sounds more more

Cr I'll do the counting seven hundred (unclear)

Ch I got five thousand

Ka Sixty three dollars

Ch Look for the Harmony carpets for the whole house with Harmony carpet Eight seven nine

Ch Seven hundred and ninety three, then for to do the whole house with Velvet carpet

Ka Velvet carpet

Ch Would be eighty dollars

Ka Eighty dollars

Cr No it wouldn't

Ka It'd be a lot more than that

Cr I'll show you what it is

Li It would be

Ka Velvet carpet

Li That's how much it is

Cr Velvet carpet

Li Cause it's more than Harmony carpet

Ka One one nine nine

Cr No, one, no

Li Would be

Ch Whoops

Ka OK now the Royal carpet, where's the Royal carpet

Cr (unclear)

Li Well of course cause it soaks through. Kay, I'll do the plush carpet

Cr Plush carpet

Ka Plush

Cr Kay, how much

Ka I'll do the

Li Twenty three

Cr Twenty

Ka I'll do the, the royal carpet

Cr Times

Ch Royal carpet, Ok I'll help you Um

L One thousand four eighty three

Ka Eighty nine

Ch Eighty nine dollars to do the whole house with Royal, with um, Harmony carpet

Li I think this is reasonable

Ch You mean, you mean you don't (unclear)

Li	Kay let's try again, sixty one times twenty three equals
Ch	The Royal carpet
Ka	Ok, hold on let me check one more, it's twenty eight bucks
Ch	One four three five
Ka	Plush sixty one, ya
Cr	Oh no, can , can I want to write it
	(unclear)
Li	[Student's name]
Cr	I like writing
Li	So do I
	(laughter)
Li	[Student's name]
Cr	Kay so it's
	(unclear)
Cr	So
Ch	You have to (unclear)
Ka	Eighty nine ya
Ch	Eighty nine dollars to do the whole house with Royal carpet
Ka	The Royal carpet
Ka	Is only eighty nine bucks
Li	You have to times it
Ka	I times what
Li	Sixty one
	(laughter)
Ch	t's twenty eight dollars and it was eighty five to do the whole carpet
Li	No, you guys I think you really screwed up
Ka	That's sort of

Li Because you've been plussing it by the wrong number I think

Ka Ya but you guys are going ahead of us, you guys got to help us, you don't just go ahead

Cr You're still plussing it

Li Well we're (unclear)

Cr ou're right where we are

Ka No, we aren't

Ch No, we're doing the Royal carpet

Ch [Student's name]

[unclear

Cr Ya, and we're doing the plush carpet So

Li Plush carpet, I just did the Plush carpet

Cr We, we haven't done the Royal yet

Ka Kay, I got one thousand seven hundred and eight

Li I got one four four three

Ka And I didn't times it

Li I timesed it

Ch One thousand seven hundred and six

Ka I don't know what I did

Li You timesed it

Ch Eight zero Oh

Ch That's zero Zero eight

Ch I got eight zero

Li [Student's name] what is your problem

Cr I forgot, I forgot that that was the right answer, sorry

Li [Student's name] come on

Ka You guys

Li You've been writing the last five million things

(unclear)

Li Aw [student's name] hna, hna

Ch One, kay so that's with the Royal carpet to do the whole, now what would it be for the Plush carpet

Ka I don't know, how much

Ch I think (unclear) should, one um

Ka Twenty three bucks times sixty one equals

(humming)

Ka One four oh three

Ch One four oh three

Ka Cause you looked on their paper

Li One four Ow

Ka You guys I think we'll be one four oh three cause she wants(unclear)

(laughter)

Ch One four oh

Li Three Woah OK, [Student's name]

Problem 2

Ka Kay, how much would it cost to use Velvet carpet in each room, show how you got your estimate

Ch Velvet carpet nineteen dollars so Velvet

Ka I get to write (unclear)

Li It's one thousand one hundred and fifty nine

Ka What

Li That's the answer (unclear) over here

Ka OK the kitchen Royal carpeted, OK the kitchen

Li You guys you're ahead of us now, how much is the Royal carpet

Ch You were ahead of us and we're not telling you how much the Royal carpet (unclear)

Li Student's name] you looked at us so we get a look at yours

Ch I looked at you and she

Ka A hundred and, one thousand seven hundred and eight nine

Ch Ya

Li OK

(unclear)

Cr Thousand

Ka Now we've just gotta estimate how much the kitchen then

Li This sucks

Ch Two dollas

Ka Zero, eight

Ch Zero eight

Li Now we can't do the estimate, you just estimate the three

Ka OK, [student's name] actual cost it says, let's put it, OK one of the kitchen, nine

Li But we have to estimate first

Ka Well why don't we do the actual cost first, that's a lot easier

Li Because, then our estimate we'll just put an estimate like one dollar under

Ka Fine

Li We can't do that

(laughter)

Ka Put it twenty bucks under

Ch Ya

Ka Easy, cinch

Li Uh

Cr I'm doing the estimate first

Li I'm doing estimate first you guys

Ka Fine, oK kitchen, there's nine squares in the kitchen, so

Cr Do you think about for the Harmony carpet

Ka Nine

Ch Guys, this is

Ka OK nine times

Li (unclear) isn't

Ka Nine times

Ka OK, nine times sixty one

Ch Ya, no, ya, no, no

Li I think it's fifty three dollars

Ka Why don't we just try it , OK

Ch Oh yeah, nine times sixty one

Ka Five hundred and forty nine

Ch Forty nine

Li I think it's five hundred and

Ch Five four nine, five four nine

Li Five hundred and thirty because like ni, nine times , nine times six is what we've got

Ka Nine times sixty one cause there 's sixty one altogether

Li Ya, but I'm rounding it so it'd be nine times six so it's fifty three

Ka Well, we'll not, we're not rounding

Ch Add four and five

Li So then just add a one

(unclear)

Ka Kay, now the actual cost

Ch he actual cost

Ka Is

Ch Is, we're doing, yeeha, um, is, what carpet, we're doing the (unclear)

Li Do it on the calculator

(unclear)

Ka Kitchen there's nine squares in the kitchen

Ch So, if what carpet are we doing

Ka I don't know

Li Times it by, I don't know

Ka Hold on guys, if we plus all the carpets

Li That's what we did before

Ka Then times nine, then maybe we will have an answer because like we don't know which carpet they're using and each carpet costs a different

Cr Eighty three altogether

K OK

Li OK

Ka So, nine

Cr Eighty three times nine eighty three

Ka Nine

Ch Nine

Li Eighty three times

Ka Times eighty three

Ch Eighty three

Ka Equals

Li Nine times

(unclear)

Ka Seven hundred and forty seven

Ch Seven hundred and forty seven

Ka I think that sort of sounds right

Li Seven hundred and forty seven

Ka Pardon me

Li Seven hundred and forty seven

Ch Forty seven Ya OK

(unclear)

Cr That's the actual answer Ya

Li Seven hundred and forty seven, [student's name]

Ka Kay now how much does the living room have, one, two, three, four

Li Family room

Ka Five, six, seven, eight, nine, ten, eleven, twelve, thirteen

Cr OK how many squares in the family room

Ka Kay there's eighteen in the living room

Cr Four, five, six, seven

Ch So guys, so eight, so that times

Ka Eighteen times eighty one

Ch Eighteen times eighty one

Ka Eighteen times eighty one

Ch Eighteen times eighty one

Li Eighteen times eighty one

Ka Woah, one thousand four hundred eighty eight

Cr No not

Li No that's kind of much

Cr Eighteen times eighty three

Ka Is that how much oh

Cr Not eighty one

Li Do you have an eraser

Ka Eighteen times eighty three

Ch Eighteen times eighty three

Ka Equals, one thousand four hundred and ninety four

Cr (unclear) my hands hurt

Ka Guys, we know what the the the the estimate

Ch The the, actual, the es, the actual, no the actual cost

Ka No, no, oh yeah that's the actual cost No

Ch Ya, because the actual cost, cause we took all the carpet

Ka No, no it isn't

Ch Ya, so what did we do last time

Li (unclear) living room

Ch We did this the the estimate first

Ka Oh no that's not wha, that, kay, that was, that's the real number

Ch Oh

Ka One hundred and forty

Ka One hundred and ninety

Ch One hundred and ninety

Ka One hundred and

Ch One four nine four, one four nine four

Cr Times (unclear)

Ka One four nine four OK

Ch Ya, now we have it

Ka	Guys, you guys are behind
Cr	My eraser
Li	It's right here
Ka	Kay guys
Cr	You have to, you have to wait for us this time
Ka	I know
Cr	Kay, what's the what's the answer to
Ka	Oh that's the family room, we're doing the living room (laughter)
Li	So what's the answer one four nine four
Ch	One four
Cr	One four nine four?
Ka	Ya
Ch	One four nine four
Ka	That cause that's the expensive one
Ch	That one four nine four is expensive carpet
Ka	OK, now the family room
Li	(unclear) estimates
Ch	The family room
Ka	Oh no no, we have to get the the estimate
Ch	Ya the estimate
Ka	OK, what did we do, I, I just remember, we
Cr	(unclear) this one, sixty (unclear)
Li	One thousand fifty There (unclear)
Ka	So, there's eighteen times
Li	Ya
Ka	Eighteen times sixty one

Ch Eighteen times sixty one

Cr Five hundred

Li Where are you guys going

Ka One hun, one thousand and ninety eight

Ch One oh nine eight

Cr What are you doing, what are you doing, what one

Ka See this is what I mean, you guys are always [student's name] slow down, we're doing the living room one kay

Li We did the living room

Cr No we estimated it

Ka Kay, what did you guys get we got one oh nine eight

Li We got one five one oh

C We, this is just an estimate

Li Ya, so who cares

Ka But we're actually

(unclear) Oooh

Li God damn

Ka [Student's name] kay now the family room uh, what should we do next

Cr The dining room

Ch Dining room

Li Dining room

Ka One, two, three, four, five, six, seven, eight, nine, ten, eleven

Li Oh [student's name]

Ka Eleven

Ch Eleven

Cr No, it

Ch That times eleven

Ka Eleven times eighty

Cr What are you doing, what are you doing

Ka Three

Ch Three

Cr Kay, what are they doing there's seventeen

Ch Nine one

Ka Thirteen

(unclear)

Cr No actual price, actual price

Ka Times

Li I got nine thirteen

C This is twelve

Ka That's what I got

Cr Twelve, not eleven

Ka Three nine (unclear) ya, ok, twelve times eighty three

Ch That's the actual

Ka Equals, what that's wrong [student's name] , definitely wrong it just equals to eighty three

Cr No but, you guys, you guys, you guys, kay listen to me, twelve, there is twelve squares in the dining room

Li OK

Ka But it equals eighty three that's that's that's

Ch Twelve

Ka Practically impossible

Li Well look how small the room is compared to the living room

Ka I know, but look at the kitchen the kitchen's even smaller than this

Li	So let's do the kitchen
Ka	And it's bigger, but it's bigger
Ch	The number is bigger and it's eighty three
Ka	This, it's nine thirteen
Ch	You could buy a chair for eighty three dollars
Ka	That's what I got
Ka	That's that's what I had too
Ch	A wooden chair
Li	Nine thirteen so let's write that down
Ch	Let alone not alone carpet your, carpet your dining room
Ka	Cause that's what I had too
Ka	[Student's name] said it wasn't right
Ch	That's it